Biointerface Research in Applied Chemistry
Platinum Open Access Journal (ISSN: 2069-5837)

Article
Volume 10, Issue 6, 2020, 6881 - 6892
https://doi.org/10.33263/BRIAC106.68816892

General Fifth M-Zagreb Polynomials of Benzene Ring Implanted in the P-Type-Surface in 2D Network

Prosanta Sarkar [1,*], Anita Pal [1]

[1] National Institute of Technology Durgapur, India
* Correspondence: prosantasarkar87@gmail.com;

Scopus Author ID 57203859853

Received: 1.05.2020; Revised: 26.05.2020; Accepted: 27.05.2020; Published: 1.06.2020

Abstract: Constitutional formulae of molecules are molecular graphs consisting of atoms as vertices and bonds between them represented as edges. The various physical, chemical, and biological properties of molecules are dependent on their molecular structures. The molecular structure is most important, not only to chemists but also to all scientists. The molecular structure descriptors or topological indices of molecules are a mathematical number or a set of selected invariants of matrices that are used to Quantitative Structure-Activity (-Property) Relationships (QSAR/QSPR) studies. In this paper, we computed some new degree-based topological indices of benzene ring implanted in the P-type-surface in the 2D network and its line subdivision of graph.

Keywords: General fifth M-Zagreb polynomials; Third Zagreb polynomial; Various degree-based topological indices; Benzene ring implanted in the P-type-surface network.

1. Introduction

In chemical graph theory, a molecular graph is a representation of the structural formula of a molecule in the form of graph structure, where atoms are represented as vertices and bonds of atoms are considered as edges. A molecular graph is a finite, simple, and connected graph. Basically, a graph is denoted by $\mathrm{G} \equiv (\mathrm{V(G)}, \mathrm{E(G)})$, where $\mathrm{V(G)}$ denote the vertex set and $\mathrm{E(G)}$ denote the edge set of G, respectively. The number of vertices in G is called the order of $\mathrm{V(G)}$ and is denoted by $|\mathrm{V(G)}|$. The number of edges in G is called the order of $E(\mathrm{G})$ and is denoted by $|E(G)|$. A subdivision graph $S(G)$ is derived from G by inserting a new vertex into each edge of G. The line graph $L(G)$, is the graph whose vertices are the edges of G and two vertices $\mathrm{e}, \mathrm{f} \in \mathrm{L(G)}$ are connected if and only if they share a common vertex in G. The degree of a vertex $\mathrm{a} \in \mathrm{G}$, is the number of neighbor vertices of a and is denoted by $\mathrm{d}_G(\mathrm{a})$. The sum of the degrees of neighbor vertices of any vertex a in G is denoted by $S_G(a)$. A molecular structure descriptor or a topological index is a real-valued function $\mathrm{f}: \mathrm{G} \to \mathbb{R}$, which maps each molecular graph to certain real numbers, and it remains invariant under graph isomorphism. In the past two decades, a large number of topological indices have been considered by some eminent researchers to utilized for relationship examination in chemistry, pharmacology, toxicology, and ecological science [1-7]. Nowadays, these indices are extensively used in building quantitative structure-property relationship (QSPR), quantitative structure-activity relationship (QSAR), and quantitative structure toxicity relationship (QSTR) [8-11]. In this work, we compute various degree-based topological indices such as fifth M −Zagreb indices and their polynomials, fifth hyper M −Zagreb indices and their polynomials, general fifth M −Zagreb indices and their polynomials, third Zagreb index or fifth M_3-Zagreb index and it is polynomial for a benzene ring implanted in P −type surface structure.

2. Materials and Methods

2.1. Various degree-based topological indices.

In [12], Graovac et al. first introduced fifth M −Zagreb indices in 2011. They defined these indices as

$$\mathrm{M}_1\,\mathrm{G}_5(\mathrm{G}) = \sum_{ab\in E(G)} (S_G(a) + S_G(b))$$

and

$$\mathrm{M}_2\,\mathrm{G}_5(\mathrm{G}) = \sum_{ab\in E(G)} (S_G(a)S_G(b)) \ .$$

In 2017, V.R. Kulli [13], generalized these indices as

$$\mathrm{M}_1^{\alpha}\,\mathrm{G}_5(\mathrm{G}) = \sum_{ab\in E(G)} (S_G(a)+S_G(b))^{\alpha}$$

and

$$\mathrm{M}_2^{\alpha}\,\mathrm{G}_5(\mathrm{G}) = \sum_{ab\in E(G)} (S_G(a)S_G(b))^{\alpha} \ .$$

In the same paper [13], he also introduced fifth hyper M −Zagreb indices as

$$\mathrm{HM}_1\,\mathrm{G}_5(\mathrm{G}) = \sum_{ab\in E(G)} (S_G(a)+S_G(b))^2$$

and

$$\mathrm{HM}_2\,\mathrm{G}_5(\mathrm{G}) = \sum_{ab\in E(G)} (S_G(a)S_G(b))^2 \,.$$

In [13], he also defined a new version of third Zagreb index or fifth M_3 −Zagreb index as

$$M_3G_5(G) = \sum_{ab\in E(G)} |S_G(a)-S_G(b)|.$$

Followed by these indices in his paper [13], he defined Zagreb polynomials as follows:

The fifth M −Zagreb polynomials are defined as

$$\mathrm{M}_1\,\mathrm{G}_5(\mathrm{G},\mathrm{x}) = \sum_{ab\in E(G)} x^{(S_G(a)+S_G(b))}$$

and

$$\mathrm{M}_2\,\mathrm{G}_5(\mathrm{G},\mathrm{x}) = \sum_{ab\in E(G)} x^{(S_G(a)S_G(b))}$$

where, x is a variable. Fifth hyper M −Zagreb polynomials are defined as

$$\mathrm{HM}_1\,\mathrm{G}_5(\mathrm{G},\mathrm{x}) = \sum_{ab\in E(G)} x^{(S_G(a)+S_G(b))^2}$$

and

$$\mathrm{HM}_2\,\mathrm{G}_5(\mathrm{G},\mathrm{x}) = \sum_{ab\in E(G)} x^{(S_G(a)S_G(b))^2} \ .$$

General fifth M −Zagreb polynomials are defined as

$$\mathrm{M}_1^{\alpha}\,\mathrm{G}_5(\mathrm{G},\mathrm{x}) = \sum_{ab \in E(G)} x^{(S_G(a)+S_G(b))^{\alpha}}$$

and

$$\mathrm{M}_2^{\alpha}\,\mathrm{G}_5(\mathrm{G},\mathrm{x}) = \sum_{ab \in E(G)} x^{(S_G(a)S_G(b))^{\alpha}}$$

where, $\alpha \in \mathbb{R}, \alpha \neq 0$ and x is a variable. The new version of third Zagreb or M_3 −Zagreb polynomial is defined as

$$\mathrm{M}_3\,\mathrm{G}_5(\mathrm{G},\mathrm{x}) = \sum_{ab \in E(G)} x^{(|S_G(a)-S_G(b)|)} .$$

Some polynomials and their corresponding topological indices were studied in [14-19].

2.2. Benzene ring implanted in P-type surface in 2D network.

The nanoscience, a period starting in 1985, when C_{60} is discovered. The branch is controlled by carbon allotropes to studying for applications in nanotechnology. Among these, nanotubes, fullerenes, graphene, diamond, and spongy nanostructures were the most studied [20-23]. In 1991, Mackay and Terrones [24], have proposed the concept of making conceivable solid carbon with three coordinated frames by tilling the infinite periodic minimal surfaces named as P and D, which are separate space into two disjoint mazes. Later Lenosky et al. [25], has proposed conceivable three-dimensional carbon solids using D tilling of 192 atoms per unit cell, a P tilling of 216 atoms per unit cell. In 1992, M.O. Keeffe et al. [26], compare the energies of these proposed structures composed of six-fold, and eightfold rings happen in the proportion 2: 3, and both have cells of just 24 atoms to those of graphite and of the icosahedral fullerene C_{60}. They named the second structure as poly benzene is seen as more steady than C_{60} energetically. Poly benzene might be depicted as a three-dimensional linkage of C_6 (benzene like) rings, thus the name poly benzene is predicted to be insulating. They manage two three dimensional frameworks of benzene, one of them is called 6.8^2P (moreover poly benzene) and has a place with space accumulate Im3m, contrasting with the P-type surface. Generally, this structure is obtained by embeddings of the hexagon-fix in the surface of the negative ebb and flow P (For more about this network, we refer our reader to [26,27]). The molecular graph G of benzene ring implanted in the P-type surface in $2D$ network and its line subdivision graph $\mathrm{L}(S(\mathrm{G}))$ is shown in Figure 1 and Figure 2, respectively.

3. Results and Discussion

In this section, we compute General fifth M −Zagreb polynomials and the new version of the third Zagreb polynomial of the molecular graph G of a two-dimensional network of benzene ring implanted in P-type-surface and its line subdivision graph $\mathrm{L}(S(\mathrm{G}))$. With the help of these two polynomials, we compute some other polynomials and their corresponding topological indices, which are mentioned earlier in this work. First, we compute general fifth M −Zagreb polynomials for G . The edge partition of graph G based on the degree sum of neighbor vertices of end vertices of each edge, is shown in Table 1. The total number of vertices and edges in G are $24\mathrm{hk}$ and $(32\mathrm{hk} - 2\mathrm{h} - 2\mathrm{k})$, respectively.

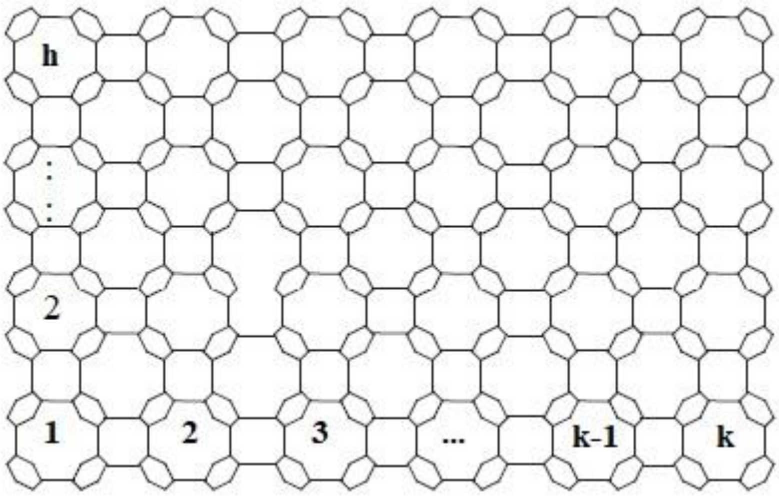

Figure 1. Two-dimensional networks of benzene ring implanted in P-type-surface

Table 1. The edge partitions with respect to the degree sum of neighbor vertices of end vertices of every edge of G.

$(S(a), S(b)): ab \in E(G)$	Total number edges
(4,5)	8
(5,5)	$(4h + 4k - 8)$
(5,7)	$(4h + 4k)$
(5,8)	$(4h + 4k - 8)$
(6,7)	$(4h + 4k)$
(6,8)	$(16hk - 12h - 12k + 8)$
(7,7)	$(2h + 2k)$
(8,8)	$(16hk - 8h - 8k)$

Theorem 1. The general fifth M_1 −Zagreb polynomial of G is given by

$$\mathrm{M}_1^{\alpha}\,\mathrm{G}_5(\mathrm{G},\mathrm{x}) = (16hk - 8h - 8k)x^{16^{\alpha}} + (16hk - 10h - 10k + 8)x^{14^{\alpha}}$$
$$+(8h + 8k - 8)x^{13^{\alpha}} + (4h + 4k)x^{12^{\alpha}} + (4h + 4k - 8)x^{10^{\alpha}} + 8x^{9^{\alpha}} \quad (1)$$

Proof. From the definition of general fifth M_1 −Zagreb polynomial index we get,

$$\mathrm{M}_1^{\alpha}\,\mathrm{G}_5(\mathrm{G},\mathrm{x}) = \sum_{ab\in E(G)} x^{(S_G(a)+S_G(b))^{\alpha}}$$
$$= \sum_{ab\in E_1(G)} x^{(4+5)^{\alpha}} + \sum_{ab\in E_2(G)} x^{(5+5)^{\alpha}} + \sum_{ab\in E_3(G)} x^{(5+7)^{\alpha}} + \sum_{ab\in E_4(G)} x^{(5+8)^{\alpha}}$$
$$+ \sum_{ab\in E_5(G)} x^{(6+7)^{\alpha}} + \sum_{ab\in E_6(G)} x^{(6+8)^{\alpha}} + \sum_{ab\in E_7(G)} x^{(7+7)^{\alpha}} + \sum_{ab\in E_8(G)} x^{(8+8)^{\alpha}}$$
$$= |E_1(G)|x^{9^{\alpha}} + |E_2(G)|x^{10^{\alpha}} + |E_3(G)|x^{12^{\alpha}} + |E_4(G)|x^{13^{\alpha}}$$
$$+|E_5(G)|x^{13^{\alpha}} + |E_6(G)|x^{14^{\alpha}} + |E_7(G)|x^{14^{\alpha}} + |E_8(G)|x^{16^{\alpha}}$$
$$= 8x^{9^{\alpha}} + (4h + 4k - 8)x^{10^{\alpha}} + (4h + 4k)x^{12^{\alpha}} + (4h + 4k - 8)\,x^{13^{\alpha}}$$
$$+(4h + 4k)\,x^{13^{\alpha}} + (16hk - 12h - 12k + 8)x^{14^{\alpha}} + (2h + 2k)x^{14^{\alpha}}$$
$$+(16hk - 8h - 8k)x^{16^{\alpha}}$$

Hence, the result follows as in equation 1.

Corollary 1. In equation 1, replacing $\alpha = 1$ and $\alpha = 2$ respectively we get the $M_1G_5(G, x)$ and $HM_1G_5(G, x)$ as follows:

(i) $M_1G_5(G, x) = (16\mathrm{hk} - 8\mathrm{h} - 8\mathrm{k})\mathrm{x}^{16} + (16\mathrm{hk} - 10\mathrm{h} - 10\mathrm{k} + 8)\mathrm{x}^{14} + (8\mathrm{h} + 8\mathrm{k} - 8)\mathrm{x}^{13} + (4h + 4k)x^{12} + (4h + 4k - 8)x^{10} + 8x^9$,

(ii) $HM_1G_5(G, x) = (16\mathrm{hk} - 8\mathrm{h} - 8\mathrm{k})\mathrm{x}^{256} + (16\mathrm{hk} - 10\mathrm{h} - 10\mathrm{k} + 8)\mathrm{x}^{196} + (8\mathrm{h} + 8\mathrm{k} - 8)\mathrm{x}^{169} + (4h + 4k)x^{144} + (4h + 4k - 8)x^{100} + 8x^{81}$.

Proposition 1. Differentiate the counting polynomial as shown in equation 1, with respect to x at $x = 1$, we get the general fifth M_1 −Zagreb index as follows:

(i) $\mathrm{M}_1^\alpha\, \mathrm{G}_5(\mathrm{G}) = (16\mathrm{hk} - 8\mathrm{h} - 8\mathrm{k})(16)^\alpha + (16\mathrm{hk} - 10\mathrm{h} - 10\mathrm{k} + 8)(14)^\alpha + (8h + 8k - 8)(13)^\alpha + (4h + 4k)(12)^\alpha + (4h + 4k -)(10)^\alpha + 8.9^\alpha$.

Corollary 2. In Proposition 1, replacing $\alpha = 1$ and $\alpha = 2$ respectively we get the $M_1G_5(G)$ and $HM_1G_5(G)$ as follows:

(i) $M_1G_5(G) = 480hk - 76h - 76k$,

(ii) $HM_1G_5(G) = 7232hk - 1680h - 1680k - 584$.

Theorem 2. The general fifth M_2 −Zagreb polynomial of G is given by

$$\mathrm{M}_2^\alpha\, \mathrm{G}_5(\mathrm{G}, \mathrm{x}) = (16\mathrm{hk} - 8\mathrm{h} - 8\mathrm{k})x^{64^\alpha} + (2\mathrm{h} + 2\mathrm{k})x^{49^\alpha} + (16\mathrm{hk} - 12\mathrm{h} - 12\mathrm{k} + 8)x^{48^\alpha} + (4\mathrm{h} + 4\mathrm{k})\, x^{42^\alpha} + (4\mathrm{h} + 4\mathrm{k} - 8)\, x^{40^\alpha} + (4\mathrm{h} + 4\mathrm{k})x^{35^\alpha} + (4\mathrm{h} + 4\mathrm{k} - 8)x^{25^\alpha} + 8x^{20^\alpha} \quad (2)$$

Proof. From the definition of general fifth M_2 −Zagreb polynomial index we get,

$$\mathrm{M}_2^\alpha\, \mathrm{G}_5(\mathrm{G}, \mathrm{x}) = \sum_{ab\in E(G)} x^{(S_G(a)S_G(b))^\alpha}$$

$$= \sum_{ab\in E_1(G)} x^{(4\times5)^\alpha} + \sum_{ab\in E_2(G)} x^{(5\times5)^\alpha} + \sum_{ab\in E_3(G)} x^{(5\times7)^\alpha} + \sum_{ab\in E_4(G)} x^{(5\times8)^\alpha} + \sum_{ab\in E_5(G)} x^{(6\times7)^\alpha} + \sum_{ab\in E_6(G)} x^{(6\times8)^\alpha} + \sum_{ab\in E_7(G)} x^{(7\times7)^\alpha} + \sum_{ab\in E_8(G)} x^{(8\times8)^\alpha}$$

$$= |E_1(G)|x^{20^\alpha} + |E_2(G)|x^{25^\alpha} + |E_3(G)|x^{35^\alpha} + |E_4(G)|x^{40^\alpha} + |E_5(G)|x^{42^\alpha} + |E_6(G)|x^{48^\alpha} + |E_7(G)|x^{49^\alpha} + |E_8(G)|x^{64^\alpha}$$

$$= 8x^{20^\alpha} + (4\mathrm{h} + 4\mathrm{k} - 8)x^{25^\alpha} + (4\mathrm{h} + 4\mathrm{k})x^{35^\alpha} + (4\mathrm{h} + 4\mathrm{k} - 8)\, x^{40^\alpha} + (4\mathrm{h} + 4\mathrm{k})\, x^{42^\alpha} + (16\mathrm{hk} - 12\mathrm{h} - 12\mathrm{k} + 8)x^{48^\alpha} + (2\mathrm{h} + 2\mathrm{k})x^{49^\alpha} + (16\mathrm{hk} - 8\mathrm{h} - 8\mathrm{k})x^{64^\alpha}$$

Which is the required result as shown in equation 2 .

Corollary 3. Replacing $\alpha = 1$ and $\alpha = 2$ respectively in equation 2, we get the $M_2G_5(G, x)$ and $HM_2G_5(G, x)$ as follows:

(i) $M_2G_5(G,x) = (16\mathrm{hk} - 8\mathrm{h} - 8\mathrm{k})x^{64} + (2\mathrm{h} + 2\mathrm{k})x^{49}$
$+(16\mathrm{hk} - 12\mathrm{h} - 12\mathrm{k} + 8)x^{48} + (4\mathrm{h} + 4\mathrm{k})\, x^{42}$
$+(4\mathrm{h} + 4\mathrm{k} - 8)\, x^{40} + (4\mathrm{h} + 4\mathrm{k})x^{35}$
$+(4\mathrm{h} + 4\mathrm{k} - 8)x^{25} + 8x^{20},$

(ii) $HM_2G_5(G,x) = (16\mathrm{hk} - 8\mathrm{h} - 8\mathrm{k})x^{4096} + (2\mathrm{h} + 2\mathrm{k})x^{2401}$
$+(16\mathrm{hk} - 12\mathrm{h} - 12\mathrm{k} + 8)x^{2304} + (4\mathrm{h} + 4\mathrm{k})\, x^{1764}$
$+(4\mathrm{h} + 4\mathrm{k} - 8)\, x^{1600} + (4\mathrm{h} + 4\mathrm{k})x^{1225}$
$+(4\mathrm{h} + 4\mathrm{k} - 8)x^{625} + 8x^{400}.$

Proposition 2. Using the first derivative of counting polynomial as shown in equation 2, at $x = 1$ we get the general fifth M_2 −Zagreb index as

(i) $M_2^{\alpha}G_5(G) = (16hk - 8h - 8k)(64)^{\alpha} + (2h + 2k)(49)^{\alpha}$
$+(16\mathrm{hk} - 12\mathrm{h} - 12\mathrm{k} + 8)(48)^{\alpha} + (4h + 4k)(42)^{\alpha}$
$+(4h + 4k - 8)(40)^{\alpha} + (4\mathrm{h} + 4\mathrm{k})(35)^{\alpha} + (4\mathrm{h} + 4\mathrm{k} - 8)(25)^{\alpha}$
$+8.(20)^{\alpha}.$

Corollary 4. From Proposition 2, we get the $M_2G_5(G)$ and $HM_2G_5(G)$ by replacing $\alpha = 1$ and $\alpha = 2$ respectively as follows:

(i) $M_2G_5(G) = 3232hk - 1862h - 1862k + 24,$

(ii) $HM_2G_5(G) = 102400hk - 34758h - 34758k + 3832.$

Theorem 3. The new version of third Zagreb polynomial of G is given by

$$M_3G_5(G,x) = (4\mathrm{h} + 4\mathrm{k} - 8)\mathrm{x}^3 + (16\mathrm{hk} - 8\mathrm{h} - 8\mathrm{k} + 8)x^2 + (4h + 4k + 8)x + (16hk - 2h - 2k - 8) \quad (3)$$

Proof. By definition of new version of third Zagreb polynomial index we get,

$$M_3G_5(G,x) = \sum_{ab\in E(G)} x^{(|S_G(a) - S_G(b)|)}$$
$$= \sum_{ab\in E_1(G)} x^{|(4-5)|} + \sum_{ab\in E_2(G)} x^{|(5-5)|} + \sum_{ab\in E_3(G)} x^{|(5-7)|} + \sum_{ab\in E_4(G)} x^{|(5-8)|}$$
$$+ \sum_{ab\in E_5(G)} x^{|(6-7)|} + \sum_{ab\in E_6(G)} x^{|(6-8)|} + \sum_{ab\in E_7(G)} x^{|(7-7)|} + \sum_{ab\in E_8(G)} x^{|(8-8)|}$$
$$= |E_1(G)|x^1 + |E_2(G)|x^0 + |E_3(G)|x^2 + |E_4(G)|x^3$$
$$+|E_5(G)|x^1 + |E_6(G)|x^2 + |E_7(G)|x^0 + |E_8(G)|x^0$$
$$= 8x + (4\mathrm{h} + 4\mathrm{k} - 8) + (4\mathrm{h} + 4\mathrm{k})x^2 + (4\mathrm{h} + 4\mathrm{k} - 8)\, x^3 + (4\mathrm{h} + 4\mathrm{k})\, x$$
$$+(16\mathrm{hk} - 12\mathrm{h} - 12\mathrm{k} + 8)x^2 + (2\mathrm{h} + 2\mathrm{k}) + (16\mathrm{hk} - 8\mathrm{h} - 8\mathrm{k})$$

Hence, the result as in equation 3.

Proposition 3. Differentiating the counting polynomial as shown in equation 3, with respect to x at $x = 1$ we get new version of third Zagreb index $M_3G_5(G)$ as

(1) $M_3G_5(G) = 32hk.$

Now we consider line subdivision graph $L(S(G))$ of two dimensional network of benzene ring implanted in P-type-surface. The edge partition with respect to the degree sum of neighbor vertices of end vertices of every edge of $L(S(G))$ is shown in Table 2. Total number of edges in $L(S(G))$ is $(88\mathrm{hk} - 10\mathrm{h} - 10\mathrm{k})$.

Table 2. The edge partitions with respect to degree sum of neighbor vertices of end vertices of every edge of $L(S(G))$.

$(S(a), S(b)): ab \in E(L(S(G)))$	**Total number edges**
(4,4)	$(4h + 4k + 4)$
(4,5)	$(8h + 8k + 4)$
(5,5)	$(8\mathrm{hk} - 4\mathrm{h} - 4\mathrm{k} + 4)$
(5,8)	$16hk$
(8,8)	$(4h + 4k)$
(8,9)	$(32hk - 8h - 8k)$
(9,9)	$(32hk - 14h - 14k)$

Theorem 4. The general fifth M_1 −Zagreb polynomial of $L(S(G))$ is given by

$$\mathrm{M}_1^\alpha\,\mathrm{G}_5(\mathrm{L}(\mathrm{S}(\mathrm{G})),\mathrm{x}) = (32hk - 14h - 14k)x^{(18)^\alpha} + (32hk - 8h - 8k)x^{(17)^\alpha}$$
$$+(4h + 4k)\, x^{(16)^\alpha} + 16hk.x^{(13)^\alpha} + (8\mathrm{hk} - 4\mathrm{h} - 4\mathrm{k} + 4)x^{(10)^\alpha}$$
$$+(8h + 8k + 4)x^{9^\alpha} + (4h + 4k + 4)x^{8^\alpha} \qquad (4)$$

Proof. From the definition of general fifth M_1 −Zagreb polynomial index we get,

$$\mathrm{M}_1^\alpha\,\mathrm{G}_5\big(\mathrm{L}(\mathrm{S}(\mathrm{G})),\mathrm{x}\big) = \sum_{ab\in E(\mathrm{L}(\mathrm{S}(\mathrm{G})))} x^{(S_G(a)+S_G(b))^\alpha}$$
$$= \sum_{ab\in E_1(\mathrm{L}(\mathrm{S}(\mathrm{G})))} x^{(4+4)^\alpha} + \sum_{ab\in E_2(\mathrm{L}(\mathrm{S}(\mathrm{G})))} x^{(4+5)^\alpha}$$
$$+ \sum_{ab\in E_3(\mathrm{L}(\mathrm{S}(\mathrm{G})))} x^{(5+5)^\alpha} + \sum_{ab\in E_4(\mathrm{L}(\mathrm{S}(\mathrm{G})))} x^{(5+8)^\alpha}$$
$$+ \sum_{ab\in E_5(\mathrm{L}(\mathrm{S}(\mathrm{G})))} x^{(8+8)^\alpha} + \sum_{ab\in E_6(\mathrm{L}(\mathrm{S}(\mathrm{G})))} x^{(8+9)^\alpha}$$
$$+ \sum_{ab\in E_7(\mathrm{L}(\mathrm{S}(\mathrm{G})))} x^{(9+9)^\alpha}$$
$$= \big|E_1\big(\mathrm{L}(\mathrm{S}(\mathrm{G}))\big)\big|x^{8^\alpha} + \big|E_2\big(\mathrm{L}(\mathrm{S}(\mathrm{G}))\big)\big|x^{9^\alpha} + \big|E_3\big(\mathrm{L}(\mathrm{S}(\mathrm{G}))\big)\big|x^{(10)^\alpha}$$
$$+\big|E_4\big(\mathrm{L}(\mathrm{S}(\mathrm{G}))\big)\big|x^{(13)^\alpha} + \big|E_5\big(\mathrm{L}(\mathrm{S}(\mathrm{G}))\big)\big|x^{(16)^\alpha}$$
$$+\big|E_6\big(\mathrm{L}(\mathrm{S}(\mathrm{G}))\big)\big|x^{(17)^\alpha} + \big|E_7\big(\mathrm{L}(\mathrm{S}(\mathrm{G}))\big)\big|x^{(18)^\alpha}$$
$$= (4h + 4k + 4)x^{8^\alpha} + (8h + 8k + 4)x^{9^\alpha} + (8\mathrm{hk} - 4\mathrm{h} - 4\mathrm{k} + 4)x^{(10)^\alpha}$$

$$+16hk.x^{(13)^{\alpha}} + (4h+4k)\,x^{(16)^{\alpha}} + (32hk-8h-8k)x^{(17)^{\alpha}}$$
$$+(32hk-14h-14k)x^{(18)^{\alpha}}$$

Which is the desired result, as shown in equation 4.

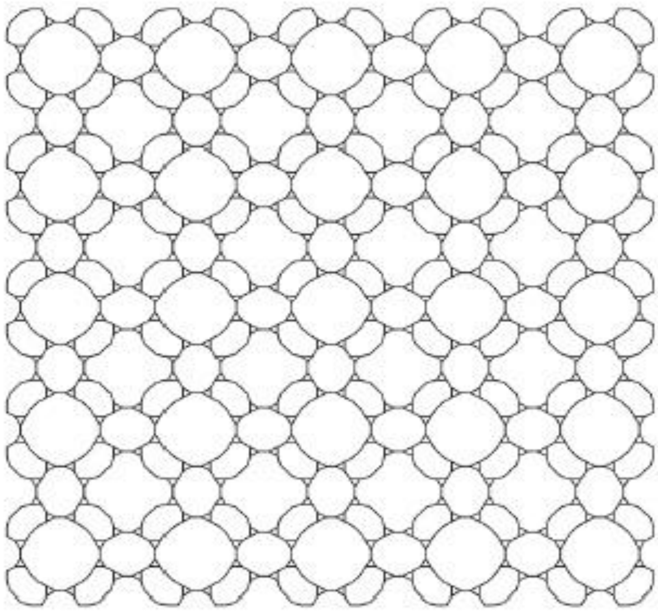

Figure 2. Line subdivision graph of benzene ring implanted in P-type-surface network

Corollary 5. Replacing $\alpha = 1$ and $\alpha = 2$, respectively in equation 4, we get $M_1G_5\big(L(S(G)),x\big)$ and $HM_1G_5\big(L(S(G)),x\big)$ as follows:

(i) $$M_1G_5\big(L(S(G)),x\big) = (32hk-14h-14k)x^{18} + (32hk-8h-8k)x^{17} +(4h+4k)\,x^{16} + 16hk.x^{13} + (8\text{hk}-4\text{h}-4\text{k}+4)x^{10} +(8h+8k+4)x^{9} + (4h+4k+4)x^{8},$$

(ii) $$HM_1G_5\big(L(S(G)),x\big) = (32hk-14h-14k)x^{324} + (32hk-8h-8k)x^{289} +(4h+4k)\,x^{256} + 16hk.x^{169} +(8\text{hk}-4\text{h}-4\text{k}+4)x^{100} + (8h+8k+4)x^{81} + (4h+4k+4)x^{64}.$$

Proposition 4. Differentiate the counting polynomial as shown in equation 4, with respect to x at $x = 1$, we get the general fifth M_1 −Zagreb index of $L\big(S(G)\big)$ as follows:

(i) $$\mathrm{M}_1^{\alpha}\,\mathrm{G}_5\big(L(S(G))\big) = (32hk-14h-14k)(18)^{\alpha} + (32hk-8h-8k)(17)^{\alpha} +(4h+4k)\,x^{(16)^{\alpha}} + 16hk.x^{(13)^{\alpha}} + (8\text{hk}-4\text{h}-4\text{k}+4)x^{(10)^{\alpha}} +(8h+8k+4)x^{9^{\alpha}} + (4h+4k+4)x^{8^{\alpha}}$$

Corollary 6. Replacing $\alpha = 1$ and $\alpha = 2$respectively in Proposition 4, we get $M_1G_5\left(L\big(S(G)\big)\right)$ and $HM_1G_5\left(L\big(S(G)\big)\right)$ as follows:

(iii) $M_1G_5\big(L\big(S(G)\big)\big) = 1408hk - 260h - 260k + 108$,

(iv) $HM_1G_5\big(L\big(S(G)\big)\big) = 23120hk - 5320h - 5320k + 980$.

Theorem 5. The general fifth M_2 −Zagreb polynomial of $L(S(G))$ is given by

$$\mathrm{M}_2^{\alpha}\,\mathrm{G}_5(\mathrm{L}(\mathrm{S}(\mathrm{G})),\mathrm{x}) = (32hk - 14h - 14k)x^{(81)^{\alpha}} + (32hk - 8h - 8k)x^{(72)^{\alpha}} + (4h + 4k)\,x^{(64)^{\alpha}} + 16hk.x^{(40)^{\alpha}} + (8\mathrm{hk} - 4\mathrm{h} - 4\mathrm{k} + 4)x^{(25)^{\alpha}} + (8h + 8k + 4)x^{(20)^{\alpha}} + (4h + 4k + 4)x^{(16)^{\alpha}} \quad (5)$$

Proof. From the definition of general fifth M_2 −Zagreb polynomial index we get,

$$\mathrm{M}_2^{\alpha}\,\mathrm{G}_5\big(\mathrm{L}\big(\mathrm{S}(\mathrm{G})\big),\mathrm{x}\big) = \sum_{ab\in E\left(\mathrm{L}\left(\mathrm{S}(\mathrm{G})\right)\right)} x^{(S_G(a)S_G(b))^{\alpha}}$$

$$= \sum_{ab\in E_1\left(\mathrm{L}\left(\mathrm{S}(\mathrm{G})\right)\right)} x^{(4\times4)^{\alpha}} + \sum_{ab\in E_2\left(\mathrm{L}\left(\mathrm{S}(\mathrm{G})\right)\right)} x^{(4\times5)^{\alpha}} + \sum_{ab\in E_3\left(\mathrm{L}\left(\mathrm{S}(\mathrm{G})\right)\right)} x^{(5\times5)^{\alpha}} + \sum_{ab\in E_4\left(\mathrm{L}\left(\mathrm{S}(\mathrm{G})\right)\right)} x^{(5\times8)^{\alpha}} + \sum_{ab\in E_5\left(\mathrm{L}\left(\mathrm{S}(\mathrm{G})\right)\right)} x^{(8\times8)^{\alpha}} + \sum_{ab\in E_6\left(\mathrm{L}\left(\mathrm{S}(\mathrm{G})\right)\right)} x^{(8\times9)^{\alpha}} + \sum_{ab\in E_7\left(\mathrm{L}\left(\mathrm{S}(\mathrm{G})\right)\right)} x^{(9\times9)^{\alpha}}$$

$$= \big|E_1\big(\mathrm{L}\big(\mathrm{S}(\mathrm{G})\big)\big)\big|x^{(16)^{\alpha}} + \big|E_2\big(\mathrm{L}\big(\mathrm{S}(\mathrm{G})\big)\big)\big|x^{(20)^{\alpha}} + \big|E_3\big(\mathrm{L}\big(\mathrm{S}(\mathrm{G})\big)\big)\big|x^{(25)^{\alpha}} + \big|E_4\big(\mathrm{L}\big(\mathrm{S}(\mathrm{G})\big)\big)\big|x^{(40)^{\alpha}} + \big|E_5\big(\mathrm{L}\big(\mathrm{S}(\mathrm{G})\big)\big)\big|x^{(64)^{\alpha}} + \big|E_6\big(\mathrm{L}\big(\mathrm{S}(\mathrm{G})\big)\big)\big|x^{(72)^{\alpha}} + \big|E_7\big(\mathrm{L}\big(\mathrm{S}(\mathrm{G})\big)\big)\big|x^{(81)^{\alpha}}$$

$$= (4h + 4k + 4)x^{(16)^{\alpha}} + (8h + 8k + 4)x^{(20)^{\alpha}} + (8\mathrm{hk} - 4\mathrm{h} - 4\mathrm{k} + 4)x^{(25)^{\alpha}} + 16hk.x^{(40)^{\alpha}} + (4h + 4k)\,x^{(64)^{\alpha}} + (32hk - 8h - 8k)x^{(72)^{\alpha}} + (32hk - 14h - 14k)x^{(81)^{\alpha}}$$

Hence the result.

Corollary 7. Putting $\alpha = 1$ and $\alpha = 2$ respectively in equation 5, we get $M_2G_5\,\big(L\big(S(G)\big),x\big)$ and $HM_2G_5\,\big(L\big(S(G)\big),x\big)$ as follows:

(i) $M_2G_5\,\big(L\big(S(G)\big),x\big) = (32hk - 14h - 14k)x^{(81)} + (32hk - 8h - 8k)x^{(72)} + (4h + 4k)\,x^{(64)} + 16hk.x^{(40)} + (8\mathrm{hk} - 4\mathrm{h} - 4\mathrm{k} + 4)x^{(25)} + (8h + 8k + 4)x^{(20)} + (4h + 4k + 4)x^{(16)}$,

(ii) $$HM_2G_5\left(L\left(S(G)\right),x\right) = (32hk - 14h - 14k)x^{(6561)} +(32hk - 8h - 8k)x^{(5184)} + (4h + 4k)\, x^{(4096)} +16hk.x^{(1600)} + (8\mathrm{hk} - 4\mathrm{h} - 4\mathrm{k} + 4)x^{(625)} +(8h + 8k + 4)x^{(400)} + (4h + 4k + 4)x^{(256)}.$$

Proposition 5. Appling the first derivative of counting polynomial as shown in equation 5, at $x = 1$, we get the general fifth M_2 −Zagreb index of $L\left(S(G)\right)$ as

(i) $$\mathrm{M}_2^{\alpha}\,\mathrm{G}_5\left(L\left(S(G)\right)\right) = (32hk - 14h - 14k)(81)^{\alpha} + (32hk - 8h - 8k)(72)^{\alpha} +(4h + 4k)\,(64)^{\alpha} + 16hk(40)^{\alpha} + (8\mathrm{hk} - 4\mathrm{h} - 4\mathrm{k} + 4)(25)^{\alpha} +(8h + 8k + 4)(20)^{\alpha} + (4h + 4k + 4)(16)^{\alpha}$$

Corollary 8. We compute the $M_2G_5\left(L\left(S(G)\right)\right)$ and $HM_2G_5\left(L\left(S(G)\right)\right)$ by replacing $\alpha = 1$ and $\alpha = 2$ respectively in Proposition 5, as follows:

(i) $M_2G_5\left(L\left(S(G)\right)\right) = 5736hk - 1330h - 1330k + 244,$

(ii) $HM_2G_5\left(L\left(S(G)\right)\right) = 406440hk - 115218h - 115218k + 5124.$

Theorem 6. The new version of third Zagreb polynomial of $L\left(S(G)\right)$ is given by

$$M_3G_5\left(L\left(S(G)\right),x\right) = 16hkx^3 + (32hk + 4)x + (40hk - 10h - 10k + 8). \quad (6)$$

Proof. From the definition of new version of third Zagreb polynomial index we get,

$$\mathrm{M}_3\,\mathrm{G}_5\left(\mathrm{L}\left(\mathrm{S(G)}\right),\mathrm{x}\right) = \sum_{ab\in E\left(\mathrm{L(S(G))}\right)} x^{(|S_G(a)-S_G(b)|)}$$

$$= \sum_{ab\in E_1\left(\mathrm{L(S(G))}\right)} x^{(|4-4|)} + \sum_{ab\in E_2\left(\mathrm{L(S(G))}\right)} x^{(|4-5|)}$$

$$+ \sum_{ab\in E_3\left(\mathrm{L(S(G))}\right)} x^{(|5-5|)} + \sum_{ab\in E_4\left(\mathrm{L(S(G))}\right)} x^{(|5-8|)}$$

$$+ \sum_{ab\in E_5\left(\mathrm{L(S(G))}\right)} x^{(|8-8|)} + \sum_{ab\in E_6\left(\mathrm{L(S(G))}\right)} x^{(|8-9|)}$$

$$+ \sum_{ab\in E_7\left(\mathrm{L(S(G))}\right)} x^{(|9-9|)}$$

$$= \left|E_1\left(\mathrm{L}\left(\mathrm{S(G)}\right)\right)\right|x^0 + \left|E_2\left(\mathrm{L}\left(\mathrm{S(G)}\right)\right)\right|x^1 + \left|E_3\left(\mathrm{L}\left(\mathrm{S(G)}\right)\right)\right|x^0$$

$$+\left|E_4\left(\mathrm{L}\left(\mathrm{S(G)}\right)\right)\right|x^3 + \left|E_5\left(\mathrm{L}\left(\mathrm{S(G)}\right)\right)\right|x^0 + \left|E_6\left(\mathrm{L}\left(\mathrm{S(G)}\right)\right)\right|x^1$$

$$+\left|E_7\left(\mathrm{L}\left(\mathrm{S}(\mathrm{G})\right)\right)\right|x^0$$

$$= (4h+4k+4)x^0 + (8h+8k+4)\,x + (8\mathrm{hk}-4\mathrm{h}-4\mathrm{k}+4)x^0 + 16hk.x^3$$

$$+(4h+4k)x^0 + (32hk-8h-8k)\,x + (32hk-14h-14k)x^0$$

Hence the result follows as in equation 6.

Proposition 6. The first derivative of counting polynomial as shown in equation 6, with respect to x at $x=1$ we get $M_3G_5\left(L\left(S(G)\right)\right)$ as

(i) $M_3G_5\left(L\left(S(G)\right)\right) = 80hk + 4.$

3. Conclusions

In this work, we computed general fifth $M-$Zagreb polynomials and the new version of third Zagreb polynomial for the molecular graph G of the two-dimensional network of benzene ring implanted in the $P-$type surface and its line subdivision graph $L\left(S(G)\right)$. Hence we computed some other polynomials and their corresponding topological indices such as fifth $M-$Zagreb polynomials and their corresponding fifth $M-$Zagreb indices, fifth hyper $M-$Zagreb polynomials and their corresponding fifth hyper $M-$Zagreb indices, M_3-Zagreb polynomial and it is corresponding M_3-Zagreb index by using our derived results. In the future study, we want to compute these indices for some chemically important molecular structures.

Funding

This research was funded by the COUNCIL OF SCIENTIFIC AND INDUSTRIAL RESEARCH (CSIR), grant number 09/973(0016)/2017-EMR-I.

Acknowledgments

We would like to thank the Editor in Chief of the journal as well as the anonymous reviewers for their careful reading of our manuscript and their many insightful comments and suggestions to improve our manuscript.

Conflicts of Interest

The authors declare no conflict of interest.

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