

Topological Properties of Para-Line Graph of Some Convex Polytopes Using Neighborhood M-Polynomial

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Abstract: The neighborhood M-polynomial is effective in recovering neighborhood degree sum based topological indices that predict different physicochemical properties and biological activities of molecular structures. Topological indices can transform the information found in molecular graphs and networks into numerical characteristics and thus make a major contribution to the study of structure-property and structure-activity relationships. In this work, the neighborhood M-polynomial of the para-line graph of some convex polytopes is obtained. From the neighborhood M-polynomial, some neighborhood degree-based topological indices are recovered. Applications of the work are described. In addition, a quantitative and graphical comparison is made.

Keywords: convex polytopes; line graph; neighborhood M-polynomial; subdivision graph; topological index.

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1. Introduction

Apart from certain mathematical sciences, the major area of implementation of graph theory is in chemistry. The great import of graph theory in chemistry has come to be widely recognized in chemical graph theory, where an outstanding area is preserved for the well-known topological indices. A topological index is a numeric quantity associated with a graph that describes the topology of the graph and is invariant under the graph isomorphism. The topological indices are used for the correlation and prediction of a very broad range of molecular properties. These range from the physical, through the chemical, the thermodynamic, the biochemical, the pharmacological, and the toxicological. The journey of topological indices was initiated by H. Wiener [1] while working on the boiling point for paraffin. Then a large number of topological indices have been developed. Recently researchers have put their attention to design topological indices based on the neighborhood degree of vertex [2-8]. By the neighborhood degree of a node, we mean the totality of degrees of all nodes that are connected to the node. The degree of a vertex is the total number of edges incident to the vertex. The computation of a certain type of topological indices using their usual definitions is laborious. To overcome this approach, many algebraic polynomials [9-11] have been introduced whose differentiation or integration or both obtained at some particular point yield topological indices. For degree-based topological indices, M-polynomial [14-16] is a very effective tool. To make the computation of neighborhood degree-based topological indices easier, present authors introduced neighborhood M-polynomial [17] whose role for <https://biointerfaceresearch.com/>

neighborhood degree-based indices is parallel to the role of the M-polynomial for degree-based indices. The subdivision [18] of a graph G , denoted by $S(G)$, is obtained by inserting an additional vertex to each edge of G . The line graph [19] of a graph G is another graph $L(G)$ such that each vertex of $L(G)$ correspond to an edge of G and two vertices of $L(G)$ are adjacent if and only if their corresponding edges share a common endpoint in G . By para-line graph [20,21] of a graph G , we mean the line graph of subdivision graph ($L(S(G))$) of G . Convex polytopes are basic geometrical objects. The beauty of their theory is now complemented by its significance for many other mathematical topics, from integration theory, algebraic topology, and algebraic geometry to linear and combinatorial optimization [22]. Also, people are paying attention to finding the metric dimension and labeling of convex polytopes [23,24]. The goal of this article is to obtain some neighborhood degree-based indices of the para-line graph of some convex polytopes using the neighborhood M-polynomial approach.

The rest of the present work is constructed as follows: Some basic definitions and formulas that are useful to obtain results are given in section 2. The materials and methods required to build the results are illustrated in section 3. Section 4 deals with the main computational results that are divided into three subsections for three para-line graphs. The results are compared numerically and graphically in section 5. We conclude the whole work in section 6.

2.Preliminaries

Throughout this article, we consider a simple connected graph. Let $V(G)$ and $E(G)$ be the vertex set and edge set of a graph G respectively. By δ_u we mean the neighborhood degree of $u \in V(G)$.

The neighborhood M-polynomial [17] of a graph G is defined as,

$$NM(G; x, y) = \sum_{i \leq j} m_{(i,j)} x^i y^j.$$

Where $m_{(i,j)}$ is the total number of edges $uv \in E(G)$ such that $\{\delta_u, \delta_v\} = \{i, j\}$. We use $NM(G)$ for $NM(G; x, y)$ for the rest of the article. The Neighborhood degree-based topological indices defined on the edge set of a graph G can be expressed as

$$I(G) = \sum_{uv \in E(G)} f(\delta_u, \delta_v),$$

where $f(\delta_u, \delta_v)$ is the function of δ_u, δ_v used in the definition of neighborhood degree-based indices. The above result can also be written as,

$$I(G) = \sum_{i \leq j} m_{(i,j)} f(i, j).$$

Now we describe some neighborhood degree-based topological indices.

The third version of the Zagreb index [3] is defined as,

$$M'_1(G) = \sum_{uv \in E(G)} (\delta_u + \delta_v).$$

The neighborhood second Zagreb index [7] is defined as,

$$M^*_2(G) = \sum_{uv \in E(G)} \delta_u \delta_v.$$

The neighborhood forgotten topological index [7] is defined as,

$$F^*_N(G) = \sum_{uv \in E(G)} (\delta_u^2 + \delta_v^2).$$

The neighborhood second modified Zagreb index [17] is defined as,

$$M^{nm}_2(G) = \sum_{uv \in E(G)} \frac{1}{\delta_u \delta_v}.$$

The neighborhood general Randić [17] index is defined as,

$$NR_\alpha(G) = \sum_{uv \in E(G)} (\delta_u \delta_v)^\alpha.$$

The third NDe index [8] is defined as,

$$ND_3(G) = \sum_{uv \in E(G)} \delta_u \delta_v (\delta_u + \delta_v).$$

The fifth NDe index [8] is defined as,

$$ND_5(G) = \sum_{uv \in E(G)} \left[\frac{\delta_u}{\delta_v} + \frac{\delta_v}{\delta_u} \right].$$

The Harmonic neighborhood index [17] is defined as,

$$NH(G) = \sum_{uv \in E(G)} \frac{2}{\delta_u + \delta_v}.$$

The neighborhood inverse sum index [17] is given by

$$NI(G) = \sum_{uv \in E(G)} \frac{\delta_u \delta_v}{\delta_u + \delta_v}.$$

The Sanskruti index [2] is defined as,

$$S(G) = \sum_{uv \in E(G)} \left(\frac{\delta_u \delta_v}{\delta_u + \delta_v - 2} \right)^3.$$

The relations of some neighborhood degree-based topological indices with the NM-polynomial are shown in Table 1.

Table 1. Derivation of some neighborhood degree-based topological indices.

Topological index	$f(x, y)$	Derivation from $NM(G)$
M'_1	$x + y$	$(D_x + D_y)(NM(G)) _{x=y=1}$
M_2^*	xy	$(D_x D_y)(NM(G)) _{x=y=1}$
F_N^*	$x^2 + y^2$	$(D_x^2 + D_y^2)(NM(G)) _{x=y=1}$
M_2^{nm}	$\frac{1}{xy}$	$(S_x S_y)(NM(G)) _{x=y=1}$
NR_α	$(xy)^\alpha$	$(D_x^\alpha D_y^\alpha)(NM(G)) _{x=y=1}$
ND_3	$xy(x+y)$	$D_x D_y (D_x + D_y)(NM(G)) _{x=y=1}$
ND_5	$\frac{x^2 + y^2}{xy}$	$(D_x S_y + S_x D_y)(NM(G)) _{x=y=1}$
NH	$\frac{2}{x+y}$	$2S_x J(NM(G)) _{x=y=1}$
NI	$\frac{xy}{x+y}$	$S_x J D_x D_y (NM(G)) _{x=y=1}$
S	$\frac{xy}{(x+y-2)^3}$	$S_x^3 Q_{-2} J D_x^3 D_y^3 (NM(G)) _{x=y=1}$

Where,

$$D_x(f(x, y)) = x \frac{\partial(f(x, y))}{\partial x}, \quad D_y(f(x, y)) = y \frac{\partial(f(x, y))}{\partial y}, \quad S_x(f(x, y)) = \int_0^x \frac{f(t, y)}{t} dt,$$

$$S_y(f(x, y)) = \int_0^y \frac{f(x, t)}{t} dt, \quad J(f(x, y)) = f(x, x), \quad Q_\alpha(f(x, y)) = x^\alpha f(x, y).$$

For some recent works on this field, readers are referred to [25-30].

3. Materials and Methods

Our main computational results include NM-polynomial and some neighborhood degree sum based topological indices of the para-line graph of convex polytopes. We utilize combinatorial computation, edge partition method, graph-theoretical tools, analytical techniques, and neighborhood degree sum counting method to derive our results. The surface plotting of NM-polynomial is made by Maple 2015.1. The results are described graphically using Matlab 2017.

4. Results and Discussion

4.1. Main results.

In this section, we obtain topological indices for the para-line graph of three types of convex polytopes using NM-polynomial. We consider the following three subsections.

4.1.1. Para-line graph of the convex polytope R_p .

A Cartesian product of a cycle graph (C_p) and a path graph of order (P_2) yields a prism graph. A p -sided antiprism is a polyhedron composed of two parallel copies of some particular p -sided polygon connected by an alternating band of the triangle. The convex polytope R_p is obtained as a combination of a prism graph and an antiprism graph, as described in [24]. The para-line graph of R_p is shown in Figure 1.

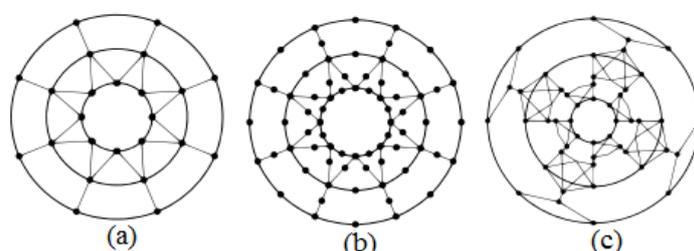


Figure 1. (a) Convex polytope R_8 (b) line graph of R_8 , and (c) para-line graph of R_8 .

Theorem 1. Let G be the para-line graph of the convex polytope R_p . Then we have

$$NM(G) = 2px^9y^9 + 2px^9y^{11} + px^{11}y^{23} + 2px^{23}y^{25} + px^{24}y^{24} + 2px^{25}y^{25} + 2px^{16}y^{16} + 4px^{16}y^{17} + px^{17}y^{17} + 2px^{17}y^{24} + 2px^{23}y^{24} + 4px^{24}y^{25}.$$

Proof. The para-line graph of the convex polytope R_p has $25p$ number of edges. Its edge set can be partitioned as follows:

Table 2. The edge partition of $L(S(R_p))$.

(δ_u, δ_v)	Cardinality	(δ_u, δ_v)	Cardinality
(9,9)	$2p$	(16,16)	$2p$
(9,11)	$2p$	(16,17)	$4p$
(11,23)	p	(17,17)	p
(23,25)	$2p$	(17,24)	$2p$
(24,24)	p	(23,24)	$2p$
(25,25)	$2p$	(24,25)	$4p$

Now using the definition of NM-polynomial and Table 2, we get

$$NM(G) = m_{(9,9)}x^9y^9 + m_{(9,11)}x^9y^{11} + m_{(11,23)}x^{11}y^{23} + m_{(23,25)}x^{23}y^{25} + m_{(24,24)}x^{24}y^{24} + m_{(25,25)}x^{25}y^{25} + m_{(16,16)}x^{16}y^{16} + m_{(16,17)}x^{16}y^{17} + m_{(17,17)}x^{17}y^{17} + m_{(17,24)}x^{17}y^{24} + m_{(23,24)}x^{23}y^{24} + m_{(24,25)}x^{24}y^{25}.$$

After putting the values of $m_{(i,j)}$'s, the required result can be obtained easily. Hence the proof. The NM-polynomial for the para-line graph of R_p is plotted in Figure 2.

Now using this NM-polynomial, we calculate some neighborhood degree-based topological indices of the para-line graph of the convex polytope R_p .

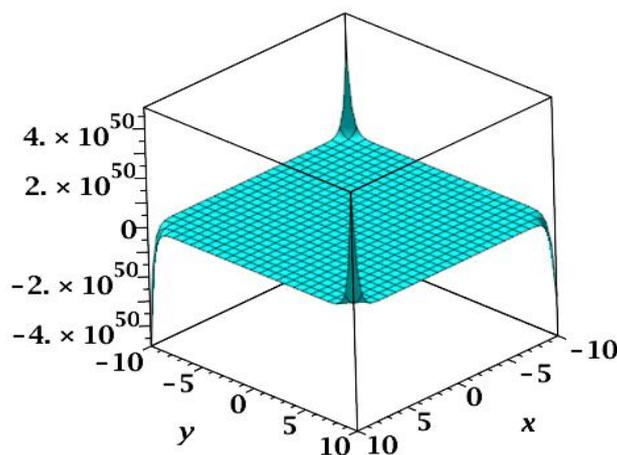


Figure 2. The NM -polynomial for the para-line graph of R_p for $p=2$.

Theorem 2. Let G be the $L(S(R_p))$. Then we have

1. $M_1'(G) = 956p$,
2. $M_2^*(G) = 9798p$,
3. $F_N^*(G) = 19864p$,
4. $M_2^{nm}(G) = 0.098p$,
5. $NR_\alpha(G) = [2(9)^{2\alpha} + 2(99)^\alpha + (253)^\alpha + 2(575)^\alpha + (576)^\alpha + 2(625)^\alpha + 2(256)^\alpha + 4(272)^\alpha + (289)^\alpha + 2(408)^\alpha + 2(552)^\alpha + 4(600)^\alpha]p$,
6. $ND_3(G) = 425884p$,
7. $ND_5(G) = 50.929p$,
8. $NH(G) = 1.458p$,
9. $NI(G) = 237.141p$,
10. $S(G) = 30355.863p$.

Proof. Let $NM(G) = f(x, y) = 2px^9y^9 + 2px^9y^{11} + px^{11}y^{23} + 2px^{23}y^{25} + px^{24}y^{24} + 2px^{25}y^{25} + 2px^{16}y^{16} + 4px^{16}y^{17} + px^{17}y^{17} + 2px^{17}y^{24} + 2px^{23}y^{24} + 4px^{24}y^{25}$.

Then we have

$$(D_x + D_y)(f(x, y)) = 36px^9y^9 + 40px^9y^{11} + 34px^{11}y^{23} + 96px^{23}y^{25} + 48x^{24}y^{24} + 100px^{25}y^{25} + 64px^{16}y^{16} + 132px^{16}y^{17} + 34px^{17}y^{17} + 82px^{17}y^{24} + 94px^{23}y^{24} + 196px^{24}y^{25},$$

$$(D_x D_y)(f(x, y)) = 162px^9y^9 + 198px^9y^{11} + 253x^{11}y^{23} + 1150px^{23}y^{25} + 576px^{24}y^{24} + 1250px^{25}y^{25} + 512px^{16}y^{16} + 1088px^{16}y^{17} + 289px^{17}y^{17} + 816px^{17}y^{24} + 1104px^{23}y^{24} + 2400px^{24}y^{25},$$

$$(D_x^2 + D_y^2)(f(x, y)) = 324px^9y^9 + 404px^9y^{11} + 650px^{11}y^{23} + 2308px^{23}y^{25} + 1152px^{24}y^{24} + 2500px^{25}y^{25} + 1024px^{16}y^{16} + 2180px^{16}y^{17} + 578px^{17}y^{17} + 1730px^{17}y^{24} + 2210px^{23}y^{24} + 4804px^{24}y^{25},$$

$$S_x S_y(f(x, y)) = \frac{2}{81} p x^9 y^9 + \frac{2}{99} p x^9 y^{11} + \frac{1}{253} x^{11} y^{23} + \frac{2}{575} p x^{23} y^{25} + \frac{1}{576} p x^{24} y^{24} + \frac{2}{625} p x^{25} y^{25} + \frac{1}{128} p x^{16} y^{16} + \frac{1}{68} p x^{16} y^{17} + \frac{1}{289} p x^{17} y^{17} + \frac{1}{204} p x^{17} y^{24} + \frac{1}{276} p x^{23} y^{24} + \frac{1}{150} p x^{24} y^{25},$$

$$D_x^\alpha D_y^\alpha(f(x, y)) = 2(9)^{2\alpha} p x^9 y^9 + 2(99)^\alpha p x^9 y^{11} + (253)^\alpha p x^{11} y^{23} + 2(575)^\alpha p x^{23} y^{25} + (576)^\alpha p x^{24} y^{24} + 2(625)^\alpha p x^{25} y^{25} + 2(256)^\alpha p x^{16} y^{16} + 4(272)^\alpha p x^{16} y^{17} + (289)^\alpha p x^{17} y^{17} + 2(408)^\alpha p x^{17} y^{24} + 2(552)^\alpha p x^{23} y^{24} + 4(600)^\alpha p x^{24} y^{25},$$

$$D_x D_y(D_x + D_y)(f(x, y)) = 2916 p x^9 y^9 + 3960 p x^9 y^{11} + 8602 p x^{11} y^{23} + 55200 p x^{23} y^{25} + 27648 p x^{24} y^{24} + 62500 p x^{25} y^{25} + 16384 p x^{16} y^{16} + 35904 p x^{16} y^{17} + 9826 p x^{17} y^{17} + 33456 p x^{17} y^{24} + 51888 p x^{23} y^{24} + 1176000 p x^{24} y^{25},$$

$$(D_x S_y + S_x D_y)(f(x, y)) = 4 p x^9 y^9 + \frac{404}{99} p x^9 y^{11} + \frac{650}{253} p x^{11} y^{23} + \frac{2308}{575} p x^{23} y^{25} + 2 p x^{24} y^{24} + 4 p x^{25} y^{25} + 4 p x^{16} y^{16} + \frac{545}{68} p x^{16} y^{17} + \frac{578}{289} p x^{17} y^{17} + \frac{865}{204} p x^{17} y^{24} + \frac{1105}{276} p x^{23} y^{24} + \frac{1201}{150} p x^{24} y^{25},$$

$$S_x J(f(x, y)) = p \left[\frac{1}{9} x^{18} + \frac{1}{10} x^{20} + \frac{1}{34} x^{34} + \frac{1}{24} x^{24} + \frac{1}{48} x^{48} + \frac{1}{25} x^{50} + \frac{1}{16} x^{32} + \frac{4}{33} x^{33} + \frac{1}{34} x^{34} + \frac{2}{41} x^{41} + \frac{2}{47} x^{47} + \frac{4}{49} x^{49} \right],$$

$$S_x J D_x D_y(f(x, y)) = p \left[9 x^{18} + \frac{99}{10} x^{20} + \frac{253}{34} x^{34} + \frac{575}{24} x^{24} + 12 x^{48} + 25 x^{50} + 16 x^{32} + \frac{1088}{33} x^{33} + \frac{17}{2} x^{34} + \frac{816}{41} x^{41} + \frac{1104}{47} x^{47} + \frac{2400}{49} x^{49} \right],$$

$$S_x^3 Q_{-2} J D_x^3 D_y^3(f(x, y)) = p [259.493 x^{16} + 332.75 x^{18} + 494.21 x^{32} + 3906.25 x^{46} + 1963.333 x^{46} + 4415.159 x^{48} + 1242.757 x^{30} + 2701.977 x^{31} + 736.62 x^{32} + 2289.901 x^{39} + 3691.558 x^{45} + 8321.855 x^{47}].$$

Using Table 1, we can easily obtain the required result.

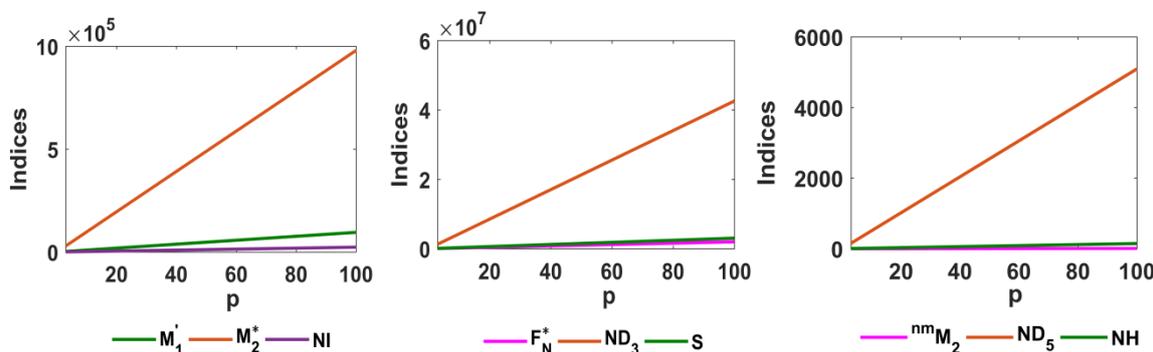


Figure 3. Graphical comparison of the indices for the para-line graph of R_p .

4.1.2. Para-line graph of the convex polytope Q_p .

The structure of convex polytope Q_p consists of 3-sided faces, 4-sided faces, 5-sided faces, and p -sided face [24]. The para line graph of Q_p is depicted in Figure 4.

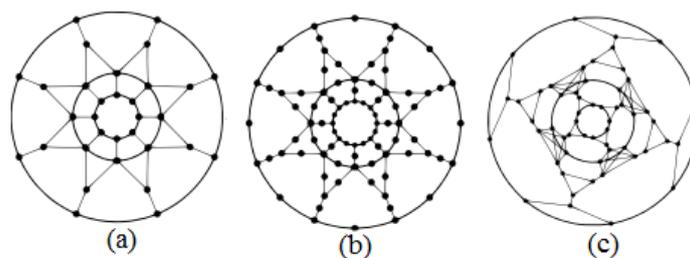


Figure 4. (a)Convex polytope Q_8 (b) line graph of Q_8 , and (c) para-line graph of Q_8 .

It contains $26p$ number of edges. Its edge set can be partitioned as follow:

Table 3. The edge partition of $L(S(Q_p))$.

(δ_u, δ_v)	Cardinality
(9,9)	$7p$
(9,11)	$4p$
(11,23)	p
(23,25)	$3p$
(24,24)	$11p$

Using this edge partition, as reported in Table 3, we obtain the following theorem as previously.

Theorem 3. Let G be the para-line graph of the convex polytope Q_p . Then we have $NM(G) = 7px^9y^9 + 4px^9y^{11} + px^{11}y^{11} + 3px^{11}y^{13} + 11px^{23}y^{23}$.

The NM-polynomial for the para-line graph of Q_p is plotted in Figure 5.

Applying Theorem 3 and Table 1, we compute the neighborhood degree-based indices for $L(S(Q_p))$ in the following theorem.

Theorem 4. Let G be the $L(S(Q_p))$. Then we have

1. $M'_1(G) = 806p$,
2. $M_2^*(G) = 7332p$,
3. $F_N^*(G) = 14692p$,
4. $M_2^{nm}(G) = 0.167p$,
5. $NR_\alpha(G) = [7(9)^{2\alpha} + 4(99)^\alpha + (121)^\alpha + 3(143)^\alpha + 11(529)^\alpha]p$,
6. $ND_3(G) = 298758p$,
7. $ND_5(G) = 52.246p$,
8. $NH(G) = 1.997p$,
9. $NI(G) = 201.175p$,
10. $S(G) = 21735.249p$.

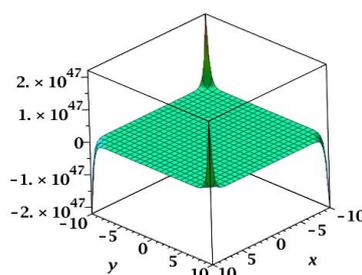


Figure 5.The NM -polynomial for the para-line graph of Q_p for $p=2$.

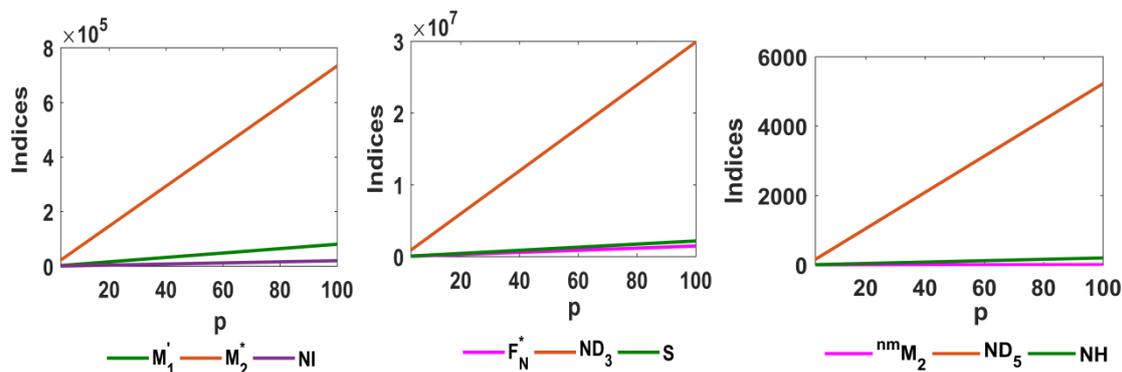


Figure 6. Graphical comparison of the indices for the para-line graph of Q_p .

4.1.3. Para-line graph of the convex polytope D_p .

The structure of convex polytope D_p consists of 5-sided faces and p -sided face as defined in [23]. The para-line graph of D_p is shown in Figure 7.

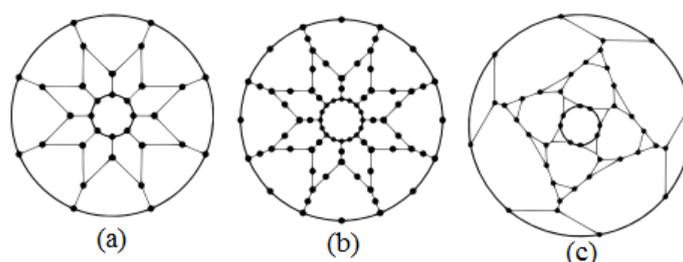


Figure 7. (a) Convex polytope D_8 (b) line graph of D_8 , and (c) para-line graph of D_8 .

The para-line graph of D_p has $18p$ edges, all of which has end vertices with neighborhood degree $(\delta_u, \delta_v) = (9,9)$. The NM polynomial of D_p is obtained in the following theorem.

Theorem 5. Let G be the para-line graph of the convex polytope D_p . Then we have $NM(G) = 18px^9y^9$.

The NM -polynomial for the para-line graph of D_p is plotted in Figure 8.

From Theorem 5, the neighborhood degree-based topological indices for the para-line graph of the convex polytope D_p are recovered in the following theorem.

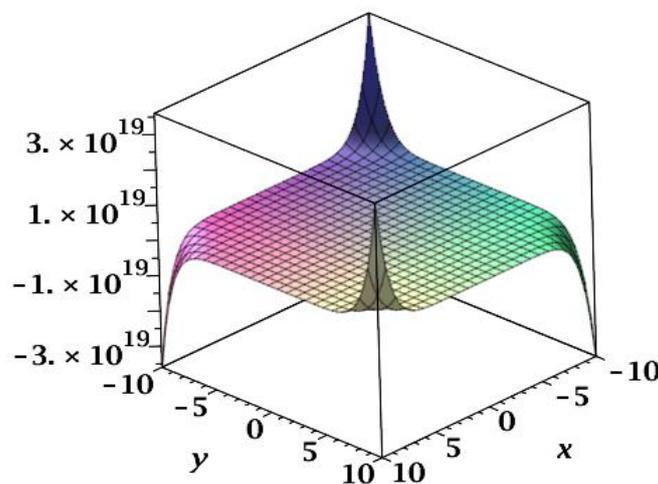


Figure 8. The NM -polynomial for the para-line graph of D_p for $p=2$.

Theorem 6. Let G be the $L(S(D_p))$. Then we have

1. $M'_1(G) = 324p$,
2. $M_2^*(G) = 1458p$,
3. $F_N^*(G) = 2916p$,
4. $M_2^{nm}(G) = 0.222p$,
5. $NR_\alpha(G) = 18(9)^{2\alpha}p$,
6. $ND_3(G) = 26244p$,
7. $ND_5(G) = 36p$,
8. $NH(G) = 2p$,
9. $NI(G) = 81p$,
10. $S(G) = 2335.434p$.

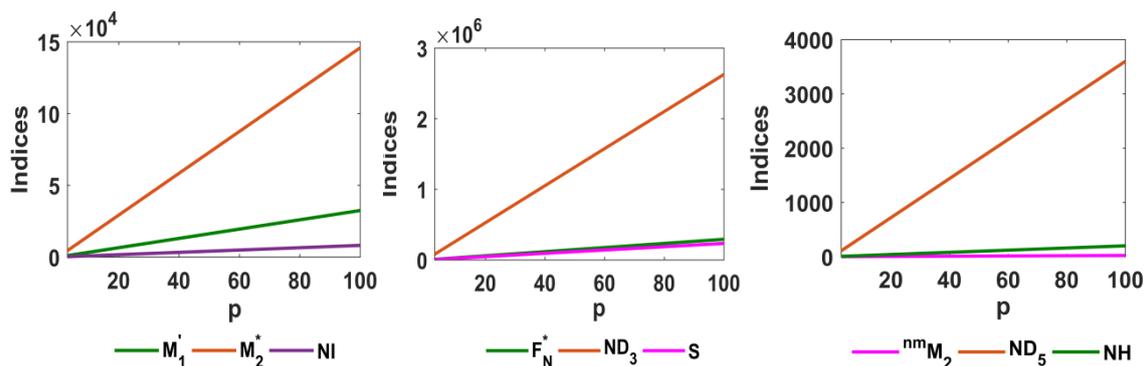


Figure 9. Graphical comparison of the indices for the para-line graph of D_p .

4.2. Comparative study.

In this section, our goal is to create a numerical and graphical comparison of different indices for the considered structures. We construct this study in two parts: firstly, all the indices taking three at a time are considered for three different structures (see Figures 3, 6, 9) and then the three para-line graphs are taken into account in one frame for different indices (see Figures 10, 11, 12, 13). Numerical values of the indices for $p = 3, 4, \dots, 10$, are listed in Tables 4-6, and their corresponding graphical representations are shown in Figures 10-12.

We consider G_1, G_2 and G_3 to represent $L(S(R_p)), L(S(Q_p))$ and $L(S(D_p))$, respectively, in Tables 4-6 and in the current discussion. In each figure, the graph parameters are considered along the horizontal axis, and the indices are along the vertical axis. Graphical representations of all indices for G_1 are shown in Figure 3. The indices for G_2 are plotted in Figure 6. The indices for G_3 are plotted in Figure 9. The third version of the Zagreb index, the neighborhood second Zagreb index, and the neighborhood forgotten topological index are listed in Table 4, and they are compared graphically in Figure 10. The neighborhood second modified Zagreb index, the third and fifth NDe indices are computed in Table 5, and the values are depicted in Figure 11. The neighborhood harmonic index, the neighborhood inverse sum index, and the Sanskriti index are computed in Table 6, and the results are compared graphically in Figure 12. The neighborhood general Randic index is plotted for a certain range of p, α in Figure 13, where the logarithm scale is used along the vertical axis to express the results fairly. It is evident from different Tables and Figures that the indices increase as the parameters of the graph increase. From figures, some comments can be drawn.

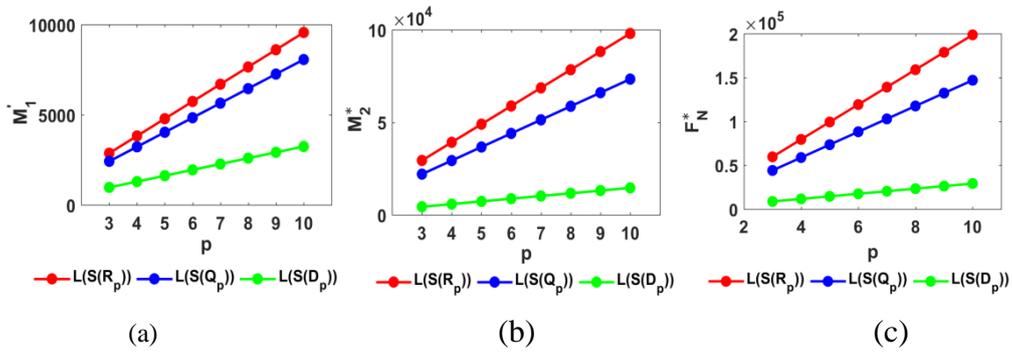


Figure 10. Graphical comparison of M_1^* , M_2^* and F_N^* for the structures under consideration.

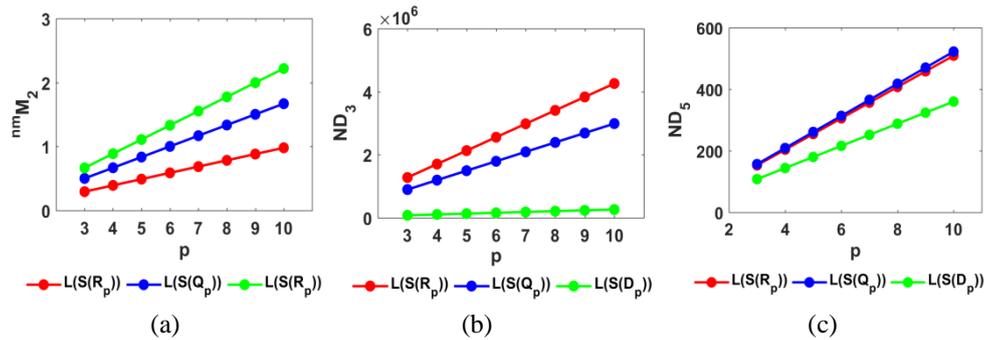


Figure 11. Graphical comparison of M_2^{nm} , ND_3 and ND_5 for the structures under consideration.

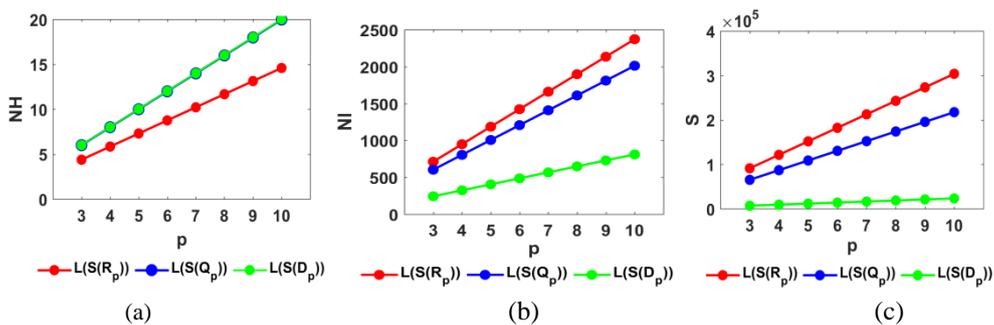


Figure 12. Graphical comparison of NH , NI , and S for the structures under consideration.

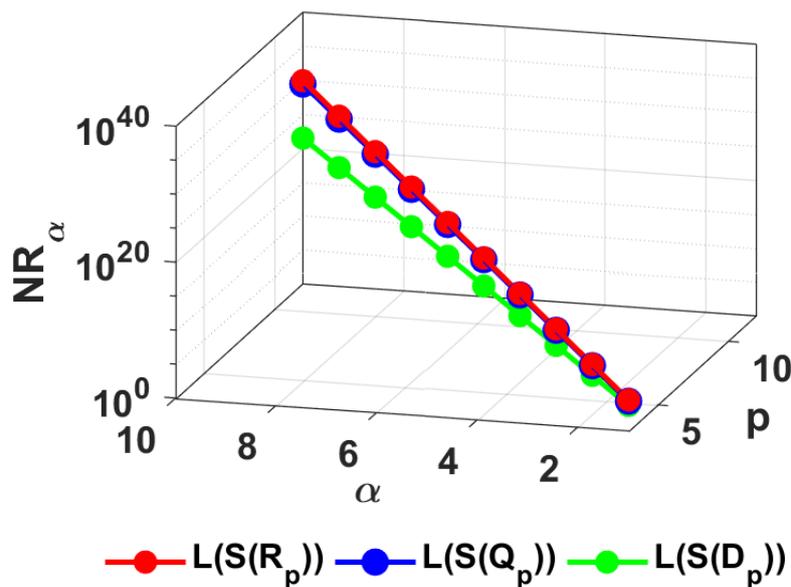


Figure 13. Graphical comparison of NR_α for the structures under consideration.

Table 4. Numerical values of M'_1, M'_2 and F_N^* for the three para-line graphs.

p	$M'_1(G_1)$	$M'_1(G_2)$	$M'_1(G_3)$	$M'_2(G_1)$	$M'_2(G_2)$	$M'_2(G_3)$	$F_N^*(G_1)$	$F_N^*(G_2)$	$F_N^*(G_3)$
3	2868	2418	972	29394	21996	4374	59592	44076	8748
4	3824	3224	1296	39192	29328	5832	79456	58768	11664
5	4780	4030	1620	48990	36660	7290	99320	73460	14580
6	5736	4836	1944	58788	43992	8748	119184	88152	17496
7	6692	5642	2268	68586	51324	10206	139048	102844	20412
8	7648	6448	2592	78384	58656	11664	158912	117536	23328
9	8604	7254	2916	88182	65988	13122	178776	132228	26244
10	9560	8060	3240	97980	73320	14580	198640	146920	29160

Table 5. Numerical values of M_2^{nm}, ND_3 and ND_5 for the three para-line graphs.

p	$M_2^{nm}(G_1)$	$M_2^{nm}(G_2)$	$M_2^{nm}(G_3)$	$ND_3(G_1)$	$ND_3(G_2)$	$ND_3(G_3)$	$ND_5(G_1)$	$ND_5(G_2)$	$ND_5(G_3)$
3	0.294	0.501	0.666	1277652	896274	78732	152.787	156.738	108
4	0.392	0.668	0.888	1703536	1195032	104976	203.716	208.984	144
5	0.49	0.835	1.11	2129420	1493790	131220	254.645	261.23	180
6	0.588	1.002	1.332	2555304	1792548	157464	305.574	313.476	216
7	0.686	1.169	1.554	2981188	2091306	183708	356.503	365.722	252
8	0.784	1.336	1.776	3407072	2390064	209952	407.432	417.968	288
9	0.882	1.503	1.998	3832956	2688822	236196	458.361	470.214	324
10	0.98	1.67	2.22	4258840	2987580	262440	509.29	522.46	360

Table 6. Numerical values of NH, NI and S for the three para-line graphs.

p	$NH(G_1)$	$NH(G_2)$	$NH(G_3)$	$NI(G_1)$	$NI(G_2)$	$NI(G_3)$	$S(G_1)$	$S(G_2)$	$S(G_3)$
3	4.374	5.991	6	711.423	603.525	243	91067.589	65205.747	7006.302
4	5.832	7.988	8	948.564	804.7	324	121423.452	86940.996	9341.736
5	7.29	9.985	10	1185.705	1005.875	405	151779.315	108676.245	11677.17
6	8.748	11.982	12	1422.846	1207.05	486	182135.178	130411.494	14012.604
7	10.206	13.979	14	1659.987	1408.225	567	212491.041	152146.743	16348.038
8	11.664	15.976	16	1897.128	1609.4	648	242846.904	173881.992	18683.472
9	13.122	17.973	18	2134.269	1810.575	729	273202.767	195617.241	21018.906
10	14.58	19.97	20	2371.41	2011.75	810	303558.63	217352.49	23354.34

In comparison with other indices, the third NDe index has the most dominating nature, whereas the neighborhood second modified Zagreb index grows slowly. The behavior of neighborhood forgotten topological index and Sanskruti index are very closed. From the vertical axis, it seems that the indices for different structures grow up in the following order: $I(G_3) < I(G_2) < I(G_1)$, except M_2^{nm}, ND_5 and NH . From Figure 11(a), it is obvious that $M_2^{nm}(G_1) < M_2^{nm}(G_2) < M_2^{nm}(G_3)$. Figure 11(c) reveals that $ND_5(G_3) < ND_5(G_1) < ND_5(G_2)$. However $ND_5(G_2)$ is very close to $ND_5(G_1)$. One can found from Figure 12(a) that $NH(G_2)$ and $NH(G_3)$ are dominating $NH(G_1)$, but it is difficult to distinguish them as their graphs are overlapped. They are very closed. When we look into Table 6, it is evident that $NH(G_2) < NH(G_3)$. For each structure, indices have the following order: $M_2^{nm} < NH < ND_5 < NI < M'_1 < M'_2 < F_N^* < S < ND_3$. Thus, we can state that all the indices behave differently in each structure discussed above.

5. Conclusions

In this work, we have considered three types of convex polytopes. Firstly we have obtained NM-polynomial of the paraline graph of the convex polytopes. Later some neighborhood degree sum based topological indices including the third version of Zagreb index, neighborhood second Zagreb index, the neighborhood was forgotten topological index, neighborhood second modified Zagreb index, neighborhood general Randic index, third NDe index, fifth NDe index, neighborhood harmonic index, neighborhood inverse sum index, and Sanskruti index for the structures under consideration had been derived using NM-polynomial.

The results are interpreted and compared graphically. This work is helpful in understanding the topology of the aforesaid structures.

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Conflicts of Interest

The authors declare no conflict of interest.

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