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Impacts of Chemical Reaction, Diffusion-Thermo and Radiation on Unsteady Natural Convective Flow past an Inclined Vertical Plate under Aligned Magnetic Field

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Abstract: In this article, diffusion-thermo, thermal radiation, and first-order chemical reaction effects are studied analytically when the aligned magnetic field set to the fluid/ the plate on the unsteady, free convective fluid passing through an inclined vertical plate by flexible surface conditions, concentration diffusion under the action of a coaxial magnetic field. The governing PDE's are derived from the physical model and transformed into dimensionless form. Then a closed-form solution is obtained using the Laplace transform method. The effects of controlling parametric quantities like M, R, Sc, Pr, Du, Gr, Gm are analyzed through graphs for fluid properties. A comparative study has been made with published results in the absence of some non-dimensional parameters for a particular case (aligned magnetic field set to the fluid) found in good agreement.

Keywords: Aligned magnetic field; Dufour effect; Thermal radiation; Chemical reaction; Inclined plate; Laplace transforms method.

Nomenclature:

a'	Absorption coefficient;	Gr	Thermal Grashof number;
T'	Temperature of the fluid (K);	R	Radiation parameter (cm^{-2});
C'	Species concentration;	Gm	Mass Grashof number;
T'_w	Temperature of the plate;	Sc	Schmidt number;
C'_w	Concentration of the plate;	$\Pr$	Prandtl number;
T'_∞	Fluid temperature away from the plate;	t	Dimensionless time (Sec);
C'_∞	Fluid Concentration away from the plate;	Nu	Nusselt number;
C	Dimensionless concentration (kg/m^3);	Sh	Sherwood number;
C_p	Specific heat at constant pressure (J/kg.K);	k	Chemical reaction (W/mK);
C_s	Concentration susceptibility;	α	Angel of inclination;
β	Volumetric coefficient of thermal expansion;	μ	Coefficient of viscosity (m^{2}s^{-1});
u_0	Velocity of the plate;	ν	Kinematic viscosity (m^{2}s^{-1});
B_0	External aligned magnetic field (A.m^2);	σ	Electric conductivity(Sm^{-1});
q_r	Radiative heat flux;	θ	Dimensionless temperature (K);

ρ	Density of the fluid (kgm^{-3});	erf	Error function;
M	Magnetic field parameter (Am^{-1});	erfc	Complementary error function;
D_m	Coefficient of mass diffusivity;		**Subscripts**
β^*	Volumetric coefficient of concentration;	w	Conditions on the wall;
g	Acceleration (m/s^2);	∞	Freestream conditions;
D_u	Diffusion-thermo effect;		

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1. Introduction

Natural or free convection is a random flow resulting from non-homogeneous fields of volumetric forces like Coriolis, MHD, gravitational, centrifugal, etc. Various researchers have been studied this phenomenon. Free/Natural convection flow has numerous practical applications and environmental situations such as chilling of electronic machinery, geothermal systems, material processing, designs connected to thermal insulation, the security of energy systems, atmospheric flows, air conditioning systems etc. Also, heat transfer applications by moving material in a moving fluid medium have a wide range of real-world applications. There are some flows on the earth whose rate is made not only by the temperature differences but also by the concentration variations. In atmosphere science, where variations among land and air temperatures can increase to complex flow shapes, buoyancy is also significant. The study of the combined transfer of heat and mass on free convection received considerable interest in many theoretical models and experimental/practical aspects because of its various applications in industry, scientific, and engineering processes. In the literature, such problems are dealt with the Newtonian/non-Newtonian fluids for many geometries such as elliptical, rectangular or cube, triangular and circular cylinders with various boundary conditions and were used different techniques such as computational, theoretical, and experimental approaches. An electrically conducting fluid flow with an external constraint of the magnetic field can be regulated. The transfer rate can also be controlled. Various industrial applications can be seen in many sciences and technology, viz nuclear cooling reactors, boundary layer control in aerodynamics, plasma studies, petroleum industries, crystal growth, etc. Hence, authors are received new attention for the study of most general contexts of MHD with the influence of the external force of the magnetic field on electrically conduction fluid.

Many practical applications like Hall accelerators, MHD power generators etc., in the engineering and industrial point of view, authors have given a considerable interest in studying Soret and Dufour effects along MHD flows when heat mass transfer occurs concurrently in moving fluids. Kao *et al*. [1] examined the solution heat transfer response of a free convective flow along with a flat plate with wall temperature discontinuity. For the plate's impulsive and uniformly accelerated motion, Rapits *et.al.* [2] studied the influence of an unvarying moving magnetic field of an electrically conducting fluid. Tokis *et al.* [3] studied the influence of a uniform moving magnetic field fixed to the fluid. Prasad *et al.* [4] have performed the effects of temperature transport properties on completely developed MHD free convective flow by considering viscous and Ohmic dissipation. On the forced convective heat transfer, Sheikholeslami *et al.* [5] investigated the outcome of a non-uniform magnetic field in a lid-driven half annuluses closure packed with Fe_3O_4-water Nanofluid. The fluid's magnetization is presumed to differ linearly with the strength of the temperature and magnetic field. Rashidi *et al.* [6] studied the heat transfer and hydrodynamic characteristics on the mixed convective nanofluid flow with sinusoidal walls under a magnetic field numerically. Shateyi *et al.* [7] examined MHD natural convective heat mass transfer flow under thermal radiation and chemical reaction over a permeable moving vertical plate with a convective boundary state using the spectral relaxation method. Khadijah *et al.* [8] analyzed normal convective viscous fluid in an annulus by ramped boundary flow based on time-dependent magnetohydrodynamic. This analysis aims to increase natural convective heat transfer, a zigzag-shaped ribs applied to the vertical, isothermal, heated surfaces. Ilias *et al.* [9] done theoretical research on the unstable aligned MHD boundary layer heat transfer nanofluid flow through an inclined plate at the interior edge. Fenuga *et al.* [10] studied the mathematical model and its solution on unstable fourth-grade MHD fluid flow using a Homotopy perturbation system under the magnetic field and suction/injection action. Prasad *et al.* [11] examined thermal radiation and absorption effects for a Kuvshinski fluid model with chemical reaction under an aligned magnetic field on the unsteady MHD flow. Effect of aligned magnetic field analyzed by Bilal *et al.* [12] for an upper convicted Maxwell fluid over an inclined stretching sheet.

Most of the authors have shown their keen interest in the combined effects of diffusion-thermo and thermo-diffusion because of its various applications in many engineering fields. In most studies, Soret and Dufour's effects are ignored due to the small magnitude of order on fluid behavior. But, in reality, Soret and Dufour's effects play a significant role when the density variations occur for the moving fluids in areas such as geosciences, petrology, hydrology etc.. They are more critical and interesting macroscopically because of their physical phenomenon in fluid mechanics. Alam *et al.* [13] have studied the effects of Dufour and Soret of a flat plate. Babu *et al.* [14] conducted a numerical study for Soret and Dufour's effects on the mixed convective electrically conducting heat transfer flow with varying fluid properties under the magnetic field. The diffusion-thermo effect is studied by Reddy *et al*. [15] in a theoretical way on the heat mass transfer flow of a viscous fluid with the aligned magnetic field. Rao *et al*. [16] published thermochemical diffusion and thermal diffusion effects using Galerkin finite element analysis on the mass and heat transfer flux.

In fluid dynamics, thermal radiation is one of the essential topics of the engineering sciences, especially in aerospace, mechanical, chemical, environmental etc. and radiated heat transfer flow plays a vital role in manufacturing industries such as wings production, steel rolling, gas turbines, nuclear reactors, various aircraft devices, furnace design, materials processing, remote sensing for astronomy, temperature measurements, food processing and numerous agricultural, as well as cryogenic engineering applications. Thermal radiation is studied numerically with various effects by (a) Rao *et al.* [17] on unsteady MHD Casson fluid from a vertical porous plate; (b) Mahato *et al.* [18] on MHD heat-absorbing unsteady Casson fluid past a flat plate; (c) Ekakitie *et al.* [19] on MHD free convection flow over an inclined plate, and (d) Lavanya *et al.* [20] on unsteady MHD viscoelastic flow past a tilted plate. In industrial and engineering sciences, under chemical reaction, combined heat and mass transfer flows have a finite number of applications like enhanced oil recovery, geothermal reservoirs, fibrous insulation, drying of porous solids, and polymers cooling and nuclear reactors, oxidation, and synthesis materials, thermal insulation etc. The study of MHD flow with an external constraint of the magnetic field can be regulated. The transfer rate can also be controlled. With this point of view, various industrial applications can be seen in many sciences and technology branches. Hence, the authors received new attention for the study of most general context of MHD with the influence of the magnetic field's external force on electrically conduction fluid. Such a MHD study has been studied for various fluids numerically under the radiation and chemical reaction effects along with the other effects, by (a) Babu *et al.* [21] varying liquid properties on the mixed convective flow from the vertical plate, (b) Krishna *et al.* [22] on the Sakiadis and the Blasius flow over a stationary flat plate, (c) Lalitha *et al.* [23] on the Oldroyd fluid flow along with a moving vertical plate, (d) Babu *et al.* [24] on the mixed convective flow past a vertical plate in a sparsely packed medium, (e) Endalew *et al.* [25] on the unstable natural convective flow past an oscillating slanted plate, (f) Sarma *et al.* [26] on the laminar mixed free-forced convective flow over a plate under a moving magnetic field, (g) Mohan *et al.* [27] on free convective Casson fluid flow past an exponentially vertical plate, (h) Omamoke *et al.* [28] on free convective flow over an inclined plate using perturbation technique by viscous dissipation and heat source, (i) Raju *et al.* [29] tested the convergence and grid independence analysis of an electrically conducting fluid past a moving plate, (j) Rao *et al.* [30] and Rama *et al.* [31] on free convective Casson fluid flow past vertical plate under the heat source. Omamoke *et al.* [32] have examined chemical reactions and thermal radiation effects on unstable blood flow through a parallel and horizontal plate in a porous and saturated medium with an inclined magnetic field.

The above literature is the motivation behind the main emphasis in studying the chemical reaction, Dufour, and radiation effects on unsteady MHD natural convective flow past an inclined plate subject to an adjustable temperature and mass diffusion under the applied aligned magnetic field. This problem is discussed in two cases when the aligned magnetic field imposed relative (i) to the fluid and (ii) to the plate. To find an exact solution of the physical model, the governed PDE's are solved using the Laplace transform technique and the boundary conditions.

2. Mathematical Formulation

Consider an unsteady, free convective, incompressible electrically conducting and radiated viscous fluid passing through an impulsively inclined plate by variable surface conditions and concentration diffusion under a uniform moving aligned magnetic field when the aligned magnetic field imposed to the fluid or the plate. Also, diffusion-thermo and first-order chemical reaction are taken into account. A uniform aligned magnetic field B_0 is imposed in the y'-direction. The x'-axis is taken along the plate facing up, and the y'-axis is taken perpendicular (normal) to the imposed aligned magnetic field. Further, it is assumed that the plate and the surrounding fluid have the same surface condition and have the same concentration at all the points in the total flow area $y' \geq 0.$ The plate has a constant speed of $u = u_0 \exp(a't')$ and the state of the plate surface increases linearly with time *t*. The mass levels around the plate increases to C'_w at $t' \geq 0$.

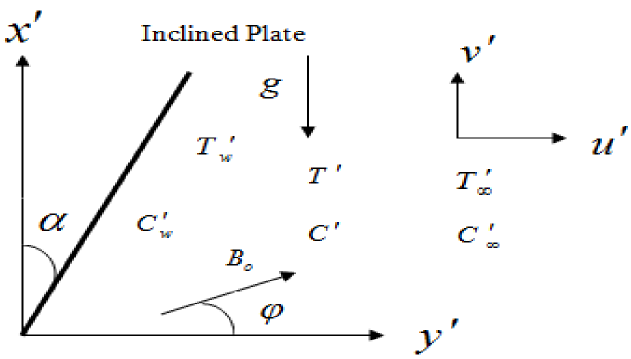

Figure 1. Geometry of the problem.

The following assumptions are made to develop a mathematical model for the free convective motion: (i) the induced magnetic field is not taken into account due to very low magnetic Reynolds number when comparing the imposed magnetic field; (ii) the viscous dissipation is ignored in the energy equation; (iii) in Boussinesque's approximation, the effects of density fluctuations and species concentration are taken into account; (iv) a thin liquid is considered with absorbing/emitting radiation in the form of a non-scattering medium; (v) all quantities are functions of y' and t' only.

The flow equations and their associated conditions, when B_0 fixed to an accelerated plate, are as follows:

$$\frac{\partial u'}{\partial t'} = v\frac{\partial^2 u'}{\partial y'^2} + g\beta^*\left(C' - C'_\infty\right)\cos\alpha + g\beta\left(T' - T'_\infty\right)\cos\alpha - \frac{\sigma B_0^2}{\rho}\left[\sin^2\varphi u' - K\,u_0\exp\left(a'_0 t'\right)\right], \quad (1)$$

$$\frac{\partial T'}{\partial t'} = \frac{k}{\rho c_p}\frac{\partial^2 T'}{\partial t'^2} + \frac{D_m K_T \rho}{\rho c_p C_s}\frac{\partial^2 C'}{\partial y'^2} - \frac{1}{\rho c_p}\frac{\partial q_r}{\partial y}, \quad (2)$$

$$\frac{\partial C'}{\partial t'} = D\frac{\partial^2 C'}{\partial y'^2} - k'\left(C' - C'_\infty\right), \quad (3)$$

$$t' \le 0: \; u' = 0, \quad T' = T'_\infty, \quad C' = C'_\infty, \quad \text{for all } \; y',$$

$$t' > 0: \; u' = u_0\exp(a't'), \; T' = T'_\infty + \left(T'_w - T'_\infty\right)At', \; C' = C'_\infty + \left(C'_w - C'_\infty\right)At', \; \text{at} \; y' = 0,$$

$$u' = 0, \; T' \to T'_\infty, \; C' \to C'_\infty, \; \text{as} \; y' \to \infty. \quad (4)$$

Here, $K = 0,1$ represents when B_0 set to the fluid or the plate respectively, a'_0 is dimensional accelerating parameter, and $A = \frac{u_0^2}{v}$.

The local radiant by Rosseland [19, 23, 28] approximation is for an optical thin gray gas is given by

$$\frac{\partial q_r}{\partial y'} = -4a^*\sigma\left(T'^4_\infty - T'^4\right). \quad (5)$$

Substituting equation (5) in (2) and rewrite, we get

$$\frac{\partial T'}{\partial t'} = \frac{k}{\rho c_p}\frac{\partial^2 T'}{\partial t'^2} + \frac{D_m K_T \rho}{\rho c_p C_s}\frac{\partial^2 C'}{\partial y'^2} - \frac{16a^*\sigma T'^3_\infty\left(T'_\infty - T'\right)}{\rho c_p}, \quad (6)$$

Introducing dimensionless quantities to express (1), (3), and (6) in non-dimensional form

$$u = \frac{u'}{u_0}, \; \theta = \frac{T' - T'_\infty}{T'_w - T'_\infty}, \; \phi = \frac{C' - C'_\infty}{C'_w - C'_\infty}, \; Sc = \frac{v}{D}, \; y = \frac{y'u_0}{v}, \; Gr = \frac{g\beta v\left(T'_w - T'_\infty\right)}{u_0^3}, \; \Pr = \frac{\mu C_\rho}{k}, \; t = \frac{t'u_0^2}{v},$$

$$M = \frac{\sigma B_0^2 v}{\rho u_0^2}, \; R = \frac{16a^* v^2 \sigma T'^3_\infty}{k u_0^2}, \; k = \frac{vk'}{u_0^2}, \; Gm = \frac{g\beta^* v\left(C'_w - C'_\infty\right)}{u_0^3}, \; Du = \frac{D_m K_T\left(C'_w - C'_\infty\right)}{c_s c_p v\left(T'_w - T'_\infty\right)}. \quad (7)$$

The resultant PDE's and associated conditions in the dimensionless form are:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + Gm\,\phi\cos\alpha + Gr\,\theta\cos\alpha - M\left(\sin^2\varphi\,u - Ke^{a_0 t}\right), \quad (8)$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial y^2} - \frac{R}{\Pr}\theta + Du\frac{\partial^2 \phi}{\partial y^2}, \quad (9)$$

$$\frac{\partial \phi}{\partial t} = \frac{1}{Sc}\frac{\partial^2 \phi}{\partial y^2} - k\phi, \quad (10)$$

$$t \le 0: \; u = 0, \; \theta = 0, \; \phi = 0, \; \text{for all} \; y,$$

$$t>0:\ u=e^{a_0 t},\ \theta=t,\ \phi=t,\ \text{at}\ \ y=0,$$
$$u\to 0,\ \theta\to 0,\ \phi\to 0\ \text{ as }\ y\to\infty. \tag{11}$$

The dimensionless PDE's (8)-(10) are solved along the BC's (11) by Laplace transform technique and finally expressed the solutions are in exponential form as follows

$$
\begin{aligned}
u(y,t)=\;&\frac{\exp(a_0 t)}{2}\left[\exp\left(y\sqrt{M_1+a_0}\right)erfc\left(\frac{y}{2\sqrt{t}}+\sqrt{(M_1+a_0)t}\right)+\exp\left(-y\sqrt{M_1+a_0}\right)erfc\left(\frac{y}{2\sqrt{t}}-\sqrt{(M_1+a_0)t}\right)\right]\\
&+A_1\left[\left(\frac{t}{2}+\frac{y}{4\sqrt{M_1}}\right)\exp\left(y\sqrt{M_1}\right)erfc\left(\frac{y}{2\sqrt{t}}+\sqrt{M_1 t}\right)+\left(\frac{t}{2}-\frac{y}{4\sqrt{M_1}}\right)\exp\left(-y\sqrt{M_1}\right)erfc\left(\frac{y}{2\sqrt{t}}-\sqrt{M_1 t}\right)\right]\\
&+\frac{A_4}{2}\exp(a_2 t)\left[\exp\left(y\sqrt{M_1+a_2}\right)erfc\left(\frac{y}{2\sqrt{t}}+\sqrt{(M_1+a_2)t}\right)+\exp\left(-y\sqrt{M_1+a_2}\right)erfc\left(\frac{y}{2\sqrt{t}}-\sqrt{(M_1+a_2)t}\right)\right]\\
&+\frac{A_2}{2}\left[\exp\left(y\sqrt{M_1}\right)erfc\left(\frac{y}{2\sqrt{t}}+\sqrt{M_1 t}\right)+\exp\left(-y\sqrt{M_1}\right)erfc\left(\frac{y}{2\sqrt{t}}-\sqrt{M_1 t}\right)\right]\\
&-\frac{A_5}{2}\exp(a_{12} t)\left[\exp\left(y\sqrt{M_1+a_{12}}\right)erfc\left(\frac{y}{2\sqrt{t}}+\sqrt{(M_1+a_{12})t}\right)+\exp\left(-y\sqrt{M_1+a_{12}}\right)erfc\left(\frac{y}{2\sqrt{t}}-\sqrt{(M_1+a_{12})t}\right)\right]\\
&+\frac{A_3}{2}\exp(-a_9 t)\left[\exp\left(y\sqrt{M_1-a_9}\right)erfc\left(\frac{y}{2\sqrt{t}}+\sqrt{(M_1-a_9)t}\right)+\exp\left(-y\sqrt{M_1-a_9}\right)erfc\left(\frac{y}{2\sqrt{t}}-\sqrt{(M_1-a_9)t}\right)\right]\\
&+\frac{A_6}{2}\exp(-a_{11} t)\left[\exp\left(y\sqrt{M_1-a_{11}}\right)erfc\left(\frac{y}{2\sqrt{t}}+\sqrt{(M_1-a_{11})t}\right)+\exp\left(-y\sqrt{M_1-a_{11}}\right)erfc\left(\frac{y}{2\sqrt{t}}-\sqrt{(M_1-a_{11})t}\right)\right]\\
&-\frac{A_7}{2}\exp(a_0 t)\left[\exp\left(y\sqrt{M_1+a_0}\right)erfc\left(\frac{y}{2\sqrt{t}}+\sqrt{(M_1+a_0)t}\right)+\exp\left(-y\sqrt{M_1+a_0}\right)erfc\left(\frac{y}{2\sqrt{t}}-\sqrt{(M_1+a_0)t}\right)\right]\\
&+\frac{A_9}{2}\left[\exp\left(y\sqrt{R}\right)erfc\left(\frac{y\sqrt{\Pr}}{2\sqrt{t}}+\sqrt{\frac{Rt}{\Pr}}\right)+\exp\left(-y\sqrt{R}\right)erfc\left(\frac{y\sqrt{\Pr}}{2\sqrt{t}}-\sqrt{\frac{Rt}{\Pr}}\right)\right]\\
&-A_{11}\left[\left(\frac{t}{2}+\frac{y\sqrt{Sc}}{4\sqrt{k}}\right)\exp\left(y\sqrt{kSc}\right)erfc\left(\frac{y\sqrt{Sc}}{2\sqrt{t}}+\sqrt{kt}\right)+\left(\frac{t}{2}-\frac{y\sqrt{Sc}}{4\sqrt{k}}\right)\exp\left(-y\sqrt{kSc}\right)erfc\left(\frac{y\sqrt{Sc}}{2\sqrt{t}}-\sqrt{kt}\right)\right]\\
&+\frac{A_{13}}{2}\exp(-a_{11}t)\left[\exp\left(y\sqrt{(k-a_{11})Sc}\right)erfc\left(\frac{y\sqrt{Sc}}{2\sqrt{t}}+\sqrt{(k-a_{11})t}\right)+\exp\left(-y\sqrt{(k-a_{11})Sc}\right)erfc\left(\frac{y\sqrt{Sc}}{2\sqrt{t}}-\sqrt{(k-a_{11})t}\right)\right]
\end{aligned}
$$

$$+\frac{A_{10}}{2}\exp(-a_9 t)\left[\begin{aligned}&\exp\left(y\sqrt{R-a_9\Pr}\right)erfc\left(\frac{y\sqrt{\Pr}}{2\sqrt{t}}+\sqrt{-a_9 t+\frac{Rt}{\Pr}}\right)\\&+\exp\left(-y\sqrt{R-a_9\Pr}\right)erfc\left(\frac{y\sqrt{\Pr}}{2\sqrt{t}}-\sqrt{-a_9 t+\frac{Rt}{\Pr}}\right)\end{aligned}\right], \tag{12}$$

$$\theta(y,t)=(1+a_5)\left[\begin{aligned}&\left(\frac{t}{2}+\frac{y\Pr}{4\sqrt{R}}\right)\exp\left(y\sqrt{R}\right)erfc\left(\frac{y\sqrt{\Pr}}{2\sqrt{t}}+\sqrt{\frac{Rt}{\Pr}}\right)\\&+\left(\frac{t}{2}-\frac{y\Pr}{4\sqrt{R}}\right)\exp\left(-y\sqrt{R}\right)erfc\left(\frac{y\sqrt{\Pr}}{2\sqrt{t}}-\sqrt{\frac{Rt}{\Pr}}\right)\end{aligned}\right]$$

$$+\frac{a_7}{2}\exp(a_2 t)\left[\begin{aligned}&\exp\left(y\sqrt{R+a_2\Pr}\right)erfc\left(\frac{y\sqrt{\Pr}}{2\sqrt{t}}+\sqrt{a_2 t+\frac{Rt}{\Pr}}\right)\\&+\exp\left(-y\sqrt{R+a_2\Pr}\right)erfc\left(\frac{y\sqrt{\Pr}}{2\sqrt{t}}-\sqrt{a_2 t+\frac{Rt}{\Pr}}\right)\end{aligned}\right]$$

$$-a_5\left[\begin{aligned}&\left(\frac{t}{2}+\frac{y\sqrt{Sc}}{4\sqrt{k}}\right)\exp\left(y\sqrt{kSc}\right)erfc\left(\frac{y\sqrt{Sc}}{2\sqrt{t}}+\sqrt{kt}\right)\\&+\left(\frac{t}{2}-\frac{y\sqrt{Sc}}{4\sqrt{k}}\right)\exp\left(-y\sqrt{kSc}\right)erfc\left(\frac{y\sqrt{Sc}}{2\sqrt{t}}-\sqrt{kt}\right)\end{aligned}\right], \tag{13}$$

$$\phi(y,t)=\left[\begin{aligned}&\left(\frac{t}{2}+\frac{y\sqrt{Sc}}{4\sqrt{k}}\right)\exp\left(y\sqrt{kSc}\right)erfc\left(\frac{y\sqrt{Sc}}{2\sqrt{t}}+\sqrt{kt}\right)\\&+\left(\frac{t}{2}-\frac{y\sqrt{Sc}}{4\sqrt{k}}\right)\exp\left(-y\sqrt{kSc}\right)erfc\left(\frac{y\sqrt{Sc}}{2\sqrt{t}}-\sqrt{kt}\right)\end{aligned}\right]. \tag{14}$$

Where, $a_1=\frac{-Du\Pr}{Sc-\Pr}$, $a_2=\frac{R-kSc}{Sc-\Pr}$, $a_3=\frac{a_1 Sc}{a_2}$, $a_4=\frac{a_1 k\,Sc}{a_2^2}$, $a_5=\frac{a_1 kSc}{a_2}$, $a_6=(1+a_5)$,

$a_7=(a_3+a_4)$, $a_8=\frac{Gr\cos\alpha}{\Pr-1}$, $a_9=\frac{R-M_1}{\Pr-1}$, $a_{10}=\frac{Gr\cos\alpha}{Sc-1}$, $a_{11}=\frac{kSc-M_1}{Sc-1}$, $a_{12}=\frac{Gm\cos\alpha}{Sc-1}$,

$a_{13}=\frac{a_8}{a_9}$, $a_{14}=\frac{a_8}{a_9^2}$, $a_{15}=\frac{a_8 a_1 Sc}{a_2 a_9}$, $a_{16}=\frac{a_1 a_8 Sc}{a_2(a_2+a_9)}$, $a_{17}=\frac{a_1 a_8 Sc}{a_9(a_2+a_9)}$, $a_{18}=\frac{a_8 a_1 kSc}{a_2 a_9}$,

$a_{19}=\frac{a_1 a_8(a_2-a_9)kSc}{a_2^2 a_9^2}$, $a_{20}=\frac{a_8 a_1 kSc}{a_2^2(a_2+a_9)}$, $a_{21}=\frac{a_8 a_1 kSc}{a_9^2(a_2+a_9)}$, $a_{22}=\frac{a_{10}a_1 Sc}{a_{11}a_{12}}$, $a_{23}=\frac{a_{10}a_1 Sc}{a_{12}(a_{11}+a_{12})}$,

$a_{24}=\frac{a_{10}a_1 Sc}{a_{11}(a_{11}+a_{12})}$, $a_{25}=\frac{a_1 a_{10} kSc}{a_{11}a_{12}}$, $a_{26}=\frac{a_1 a_{10} kSc(a_{12}-a_{11})}{a_{11}^2 a_{12}^2}$, $a_{27}=\frac{a_{10}a_1 kSc}{a_{12}^2(a_{11}+a_{12})}$, $a_{28}=\frac{a_{10}a_1 kSc}{a_{11}^2(a_{11}+a_{12})}$,

$a_{29}=\frac{a_{12}}{a_{11}}$, $a_{30}=\frac{a_{12}}{a_{11}^2}$, $a_{31}=\frac{MK}{M_1+a_0}$, $A_1=(a_{13}-a_{18}+a_{25}+a_{29})$, $A_2=(-a_{14}-a_{15}+a_{19}+a_{22}-a_{26}-a_{30})$,

$A_3=(a_{14}+a_{17}-a_{21})$, $A_4=(a_{16}+a_{20})$, $A_5=(a_{23}+a_{27})$, $A_6=(-a_{24}+a_{28}+a_{30})$, $A_7=a_{31}$,

$A_8=(-a_{13}+a_{18})$, $A_9=(a_{14}+a_{15}-a_{19})$, $M_1=M\,Sin^2\varphi$ $A_{10}=(-a_{14}-a_{17}+a_{21})$, $A_{11}=(a_{25}+a_{29})$,

$A_{12}=(-a_{22}+a_{26}+a_{30})$, $A_{13}=(a_{24}-a_{28}-a_{30})$.

The Nusselt and Sherwood number's in the non-dimensional form are given by:

$$Nu=a_6\left[t\sqrt{R}\,erf\sqrt{\frac{Rt}{\Pr}}+\sqrt{\frac{t\Pr}{\pi}}\exp\left(-\frac{Rt}{\Pr}\right)+\frac{\Pr}{2\sqrt{R}}erf\sqrt{\frac{Rt}{\Pr}}\right]$$

$$-\left[\sqrt{\frac{\Pr}{\pi t}}\exp\left(-\frac{Rt}{\Pr}\right)+\sqrt{R}\,erf\sqrt{\frac{R}{\Pr}t}\right]+a_7\left[\sqrt{\frac{Sc}{\pi t}}\exp(-kt)+\sqrt{kSc}\,erf\sqrt{kt}\right]$$

$$+a_7\left[\sqrt{\frac{\Pr}{\pi t}}\exp\left(-\frac{Rt}{\Pr}\right)+\exp(a_2 t)\sqrt{R+a_2\Pr}\,erf\sqrt{\left(\frac{R}{\Pr}+a_2\right)t}\right]$$

$$-a_5\left[t\sqrt{kSc}\,erf\left(\sqrt{kt}\right)+\sqrt{\frac{tSc}{\pi}}\exp(-kt)+\frac{\sqrt{Sc}}{2\sqrt{k}}exf\left(\sqrt{kt}\right)\right], \tag{15}$$

$$Sh=t\sqrt{kSc}\,erf\sqrt{kt}+\sqrt{\frac{t\Pr}{\pi}}\exp\left(-kt\right)+\frac{\sqrt{Sc}}{2\sqrt{k}}erf\sqrt{kt}. \tag{16}$$

3. Results and Discussion

To understand the mathematical model's physics, an analytical study has been conducted, presenting the results in the form of plots. The fluid properties such as velocity, concentration, and the rates of heat and mass transfer of the fluid as a result of the difference in the most significant non-dimensional parameters like magnetic parameter (M), Dufour number (Du), radiation parameter (R), chemical reaction parameter (k), Schmidt number (Sc), Prandtl number (Pr), thermal and solutal Grashof numbers $(Gr\&Gm)$ in Figures 2-11. To verify whether the obtained results are correct or not, a comparative study has been done when B_0 fixed to the fluid $(K=0)$ with the existing literature in the absence of some dimensionless quantities and found in good agreement.

The variations of *M*, *Gm*, *Du*, *Gr*, *Sc*, and *t* on the velocity profiles are presented from figures 2-7 for an exponentially accelerated plate in the presence of radiation and chemical reaction $K=0$ &1 by fixing other parameters *Sc*=2.01, *R*=10, *Pr*=0.71, *t*=0.4, *k*=1, *Du*=0.03, $\alpha=60^o, \varphi=30^o$, *M*=5, a_0=0.5. The outcome of the magnetic field on velocity profiles for $K=0$ and $K=1$are discussed in Figures 2 and 3. It is observed that the velocity profiles reduce with an increase in the magnetic parameter when cooling and heating the plate. Applying an inclined magnetic field to an electric conductor fluid causes a drag force (Lorentz force). The effect of Dufour number on the velocity profiles is depicted in Figures. 4 and 5 for $K=0$ and $K=1$. It is observed that, in the case of plate cooling, the velocity reduces with the increase of the Dufour number, the trend is reversed when heating the plate. Figures 6 and 7 disclose the effects of time *t* on the velocity profiles for $K=0$ and $K=1$. From these figures, it is seen that the velocity profiles rise as the plate is cooled and heated.

The variation of different physical quantities on temperature profiles are shown in Figures 8-10. Figure 8 shows the influence of Dufour number on temperature distributions, noticing that diffusional thermal effects significantly impact the liquid's temperature, and the fluid temperature decreases as the Dufour number increases. The effect of the thermal radiation (R) parameter on the temperature is shown in Figure 9. From there, it is seen that the radiation parameter reduces the temperature of the liquid. The variations of Prandtl number (Pr) illustrated on the temperature field in figure 10 reveal that as Pr increases, the temperature decreases. This is because the thickness of the thermal boundary layer decreases. The variations of Schmidt number (Sc), by considering hydrogen $(Sc=0.22)$, water $(Sc=0.60)$, and oxygen $(Sc=0.75)$ and the time (t) on concentration profiles are depicted in Figure 11. Figure 11 shows that the concentration regularly rises for hydrogen and accumulates oxygen in contrast to the water vapor. Thus water vapor can be used to maintain the concentration profiles, and hydrogen can be used to maintain a strong concentration field.

The concentration profile decreases with an increase in *k*, as it is observed in Figure 12. Figure 13 shows the Nusselt number versus time t.

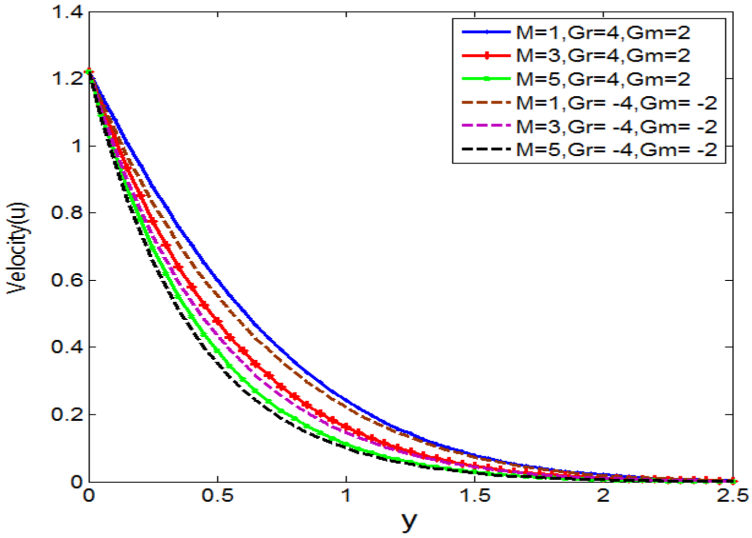

Figure 2. When *K*=0, the velocity variations for distinct *M* values.

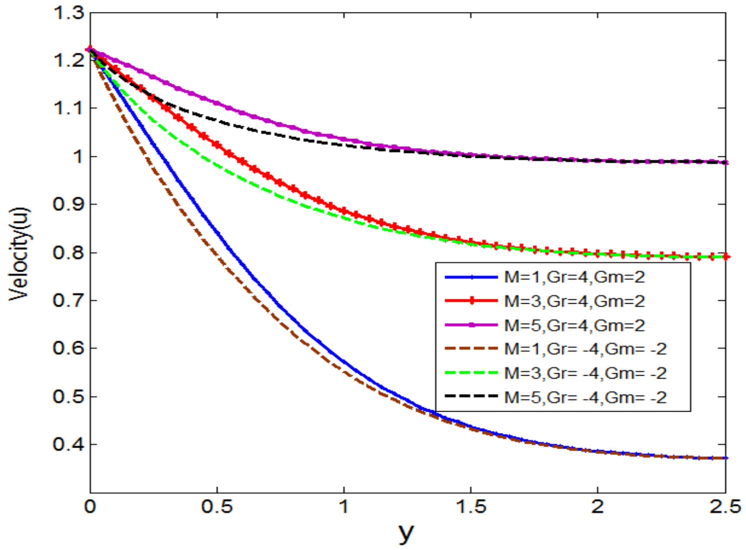

Figure 3. When *K*=1, the velocity variations for distinct *M* values.

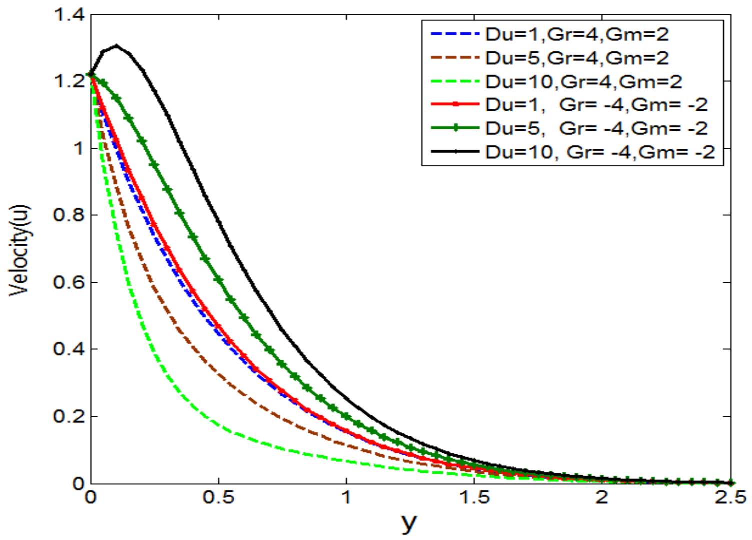

Figure 4. When *K*=0, the velocity variations for distinct *Du* values.

The Nusselt number rises with increasing R values for both air $(\Pr = 0.71)$ and water $(\Pr = 7.0)$, and it is seen that the rate of heat transfer with the water plate is high compared to air. This is because modest values $\Pr$ are equivalent to an increase in heat transmission and therefore, can spread much faster than higher Pr values. Figure 14 represents the Sherwood number versus time t. The Sherwood number rises with the increase of Sc.

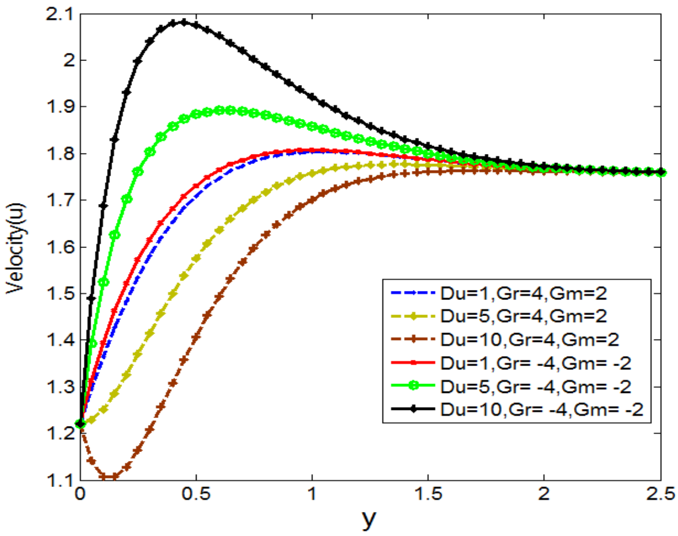

Figure 5. When K=1, the velocity variations for distinct Du values.

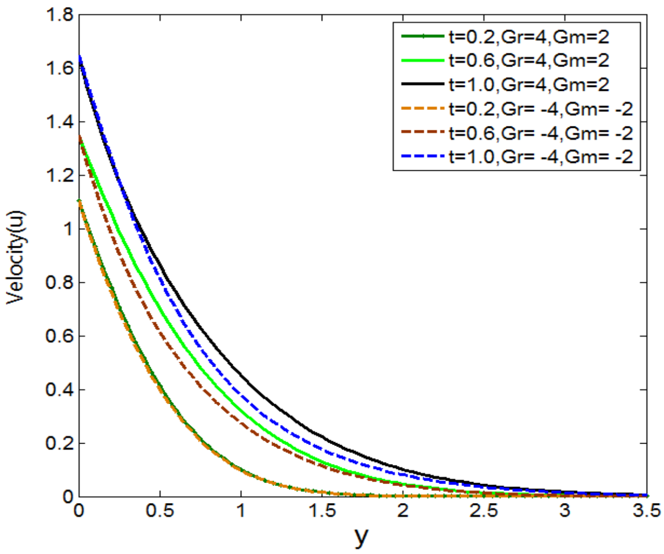

Figure 6. When K=0, the velocity variations for distinct t values.

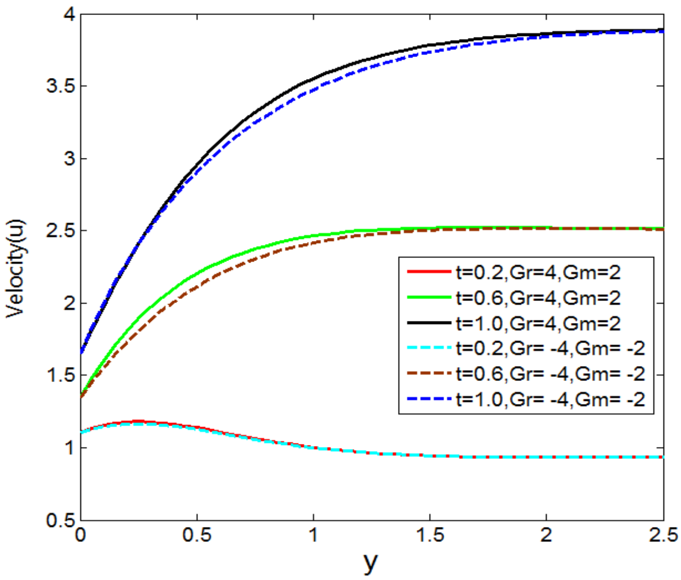

Figure 7. When K=1, the velocity variations for distinct t values.

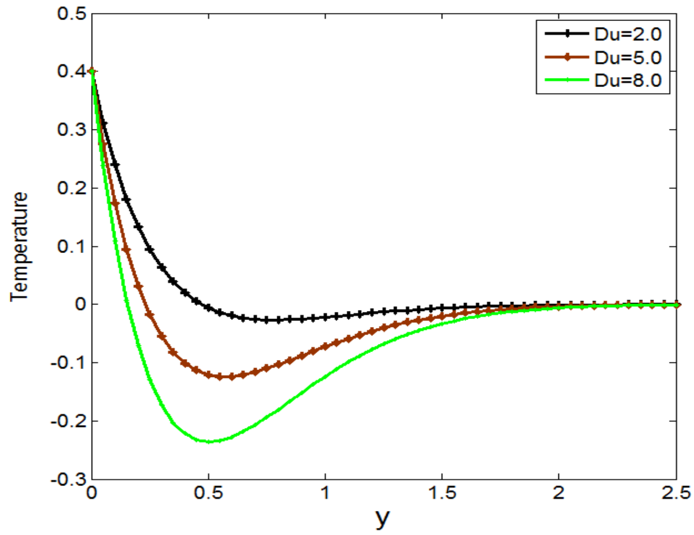

Figure 8. Temperature variations for distinct *Du* values, when $Sc = 2.1, R = 10, \Pr = 0.71, t = 0.4, k = 0.5.$

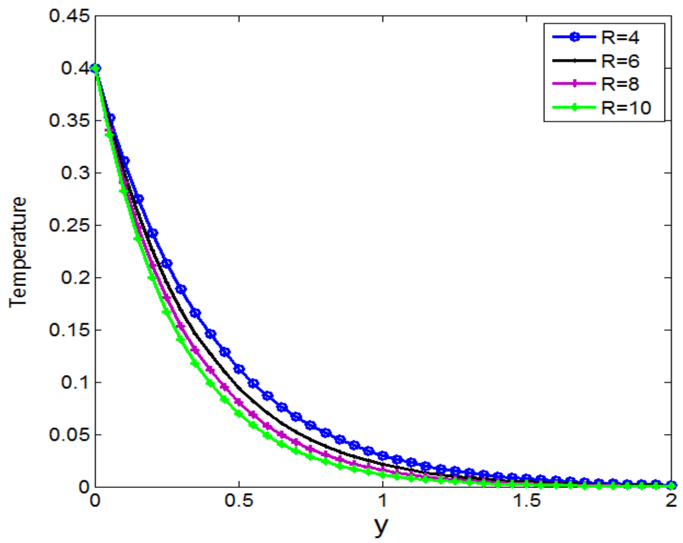

Figure 9. Temperature variations for distinct *R* values, when $Sc = 2.1, Du = 0.03, \Pr = 0.71, t = 0.4, k = 0.5.$

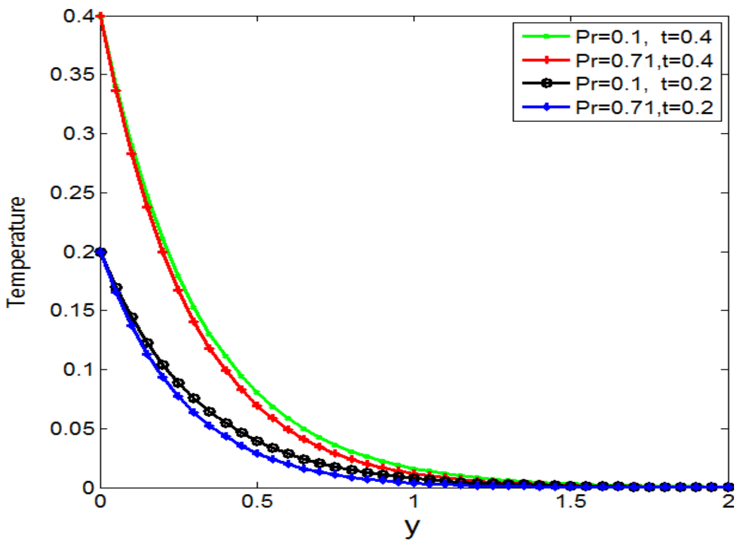

Figure 10. Temperature variations for distinct *Pr* and *t* values,

when $Sc = 2.1,\ R = 10,\ Du = 0.03,\ k = 0.5.$

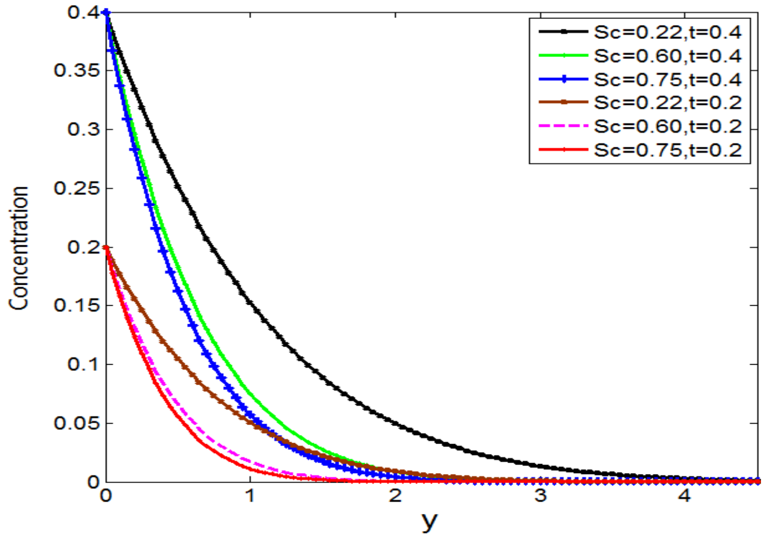

Figure 11. Concentration variations for distinct *Sc* and *t* values, when *k*= 0.5.

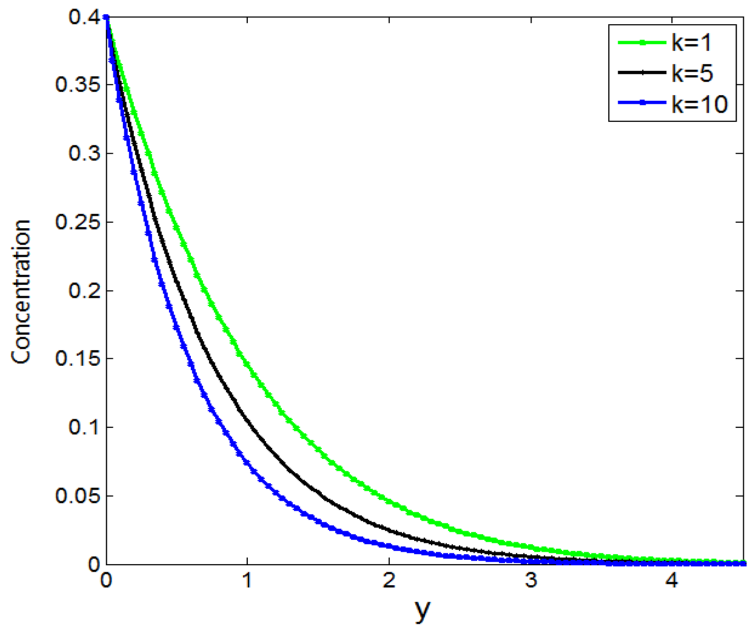

Figure 12. Concentration variations for distinct *k* values, when $Sc = 0.22,\ t = 0.4.$

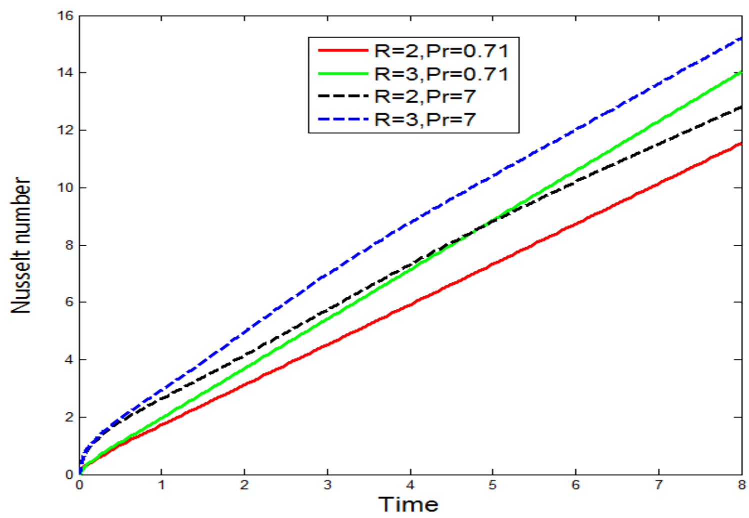

Figure 13. Nusselt number for distinct *R* and *Pr* values, when $Sc = 2.1,\ Du = 0.03,\ R = 10, k = 0.5.$

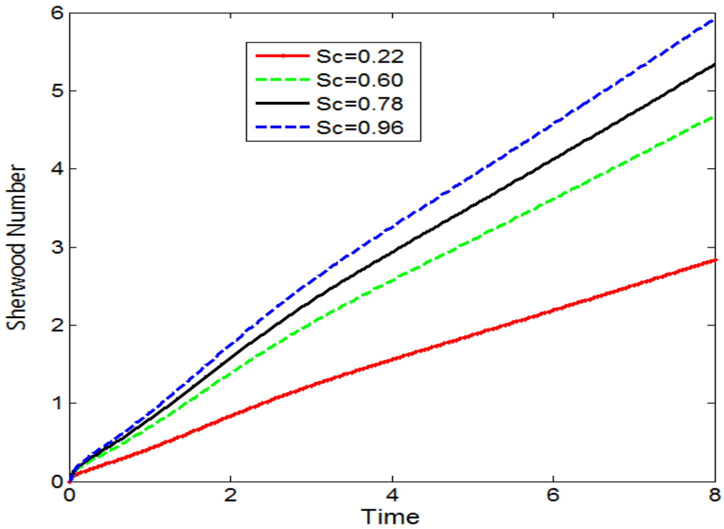

Figure 14. Sherwood number for distinct *Sc* values, when $\mathrm{Pr} = 0.71$ and $k = 0.5$.

Table 1 shows the change in the velocity profile for various values of R. It is observed that velocity reduces when heating and cooling the plate for $K = 0$. The opposite trend is seen for $K = 1$.

Table 1. Velocity distributions for various values of *R*, when $K = 0, 1$.

		$K=0$			$K=1$		
	y	***R*=1**	***R*=5**	***R*=10**	***R*=1**	***R*=5**	***R*=10**
Cooling of the plate	**0.0000**	1.2214	1.2214	1.2214	1.2214	1.2214	1.2214
	0.2000	0.9385	0.9335	0.9290	1.5323	1.5272	1.5228
	0.4000	0.6995	0.6917	0.6856	1.7046	1.6968	1.6907
	0.6000	0.5055	0.4972	0.4912	1.7890	1.7806	1.7746
	0.8000	0.3540	0.3464	0.3413	1.8209	1.8133	1.8083
	1.0000	0.2398	0.2336	0.2298	1.8244	1.8182	1.8143
Heating of the plate	**0.0000**	1.2214	1.2214	1.2214	1.2214	1.2214	1.2214
	0.2000	0.8742	0.8793	0.8837	1.4679	1.4730	1.4774
	0.4000	0.6211	0.6289	0.6350	1.6262	1.6339	1.6401
	0.6000	0.4353	0.4437	0.4497	1.7187	1.7271	1.7331
	0.8000	0.2991	0.3067	0.3118	1.7661	1.7737	1.7787
	1.0000	0.2005	0.2067	0.2105	1.7850	1.7912	1.7950

4. Conclusions

In this article, the effects of diffusion-thermo, thermal radiation, and chemical reaction are studied on unsteady, free convective MHD fluid passing through an inclined plate with variable surface conditions and concentration diffusion change under a coaxial magnetic field, when $K = 0$ and $K = 1$. A theoretical study has been conducted using Laplace transform method without any restriction in order to get the solution of the problem. The fluid properties for various flow controlling parameters like *M*, *R*, *Sc*, *Pr*, *Du*, *Gr*, *Gm* are analyzed through graphs. A comparison is made with published results in the absence of some non-dimensional parameters. The following conclusions are drawn to the present study: (i) in case of cooling of the plate, the velocity decreases with an increasing *M* and *Du* values, and an opposite trend is observed in the case of heating the plate; (ii) the temperature profiles decrease significantly with increasing values of *Du*, *Pr*, and *R*; (iii) the concentration profiles decrease significantly with increasing values of *Sc* and *k*; (iv) the Nusselt number increases with increasing R values, and Sherwood number increases with the increase of *Sc*.

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Conflicts of Interest

The authors declare no conflict of interest.

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