

# Structural and Defect Properties of $\text{LiTi}_2(\text{PO}_4)_3$

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**Abstract:**  $\text{LiTi}_2(\text{PO}_4)_3$  is an attractive electrolyte material in Li-ion batteries' application due to its high ionic conductivity and high chemical stability. Here we employ atomistic simulation based on the classical pair potentials to examine the intrinsic defect processes, Li-ion migration, and solution of various dopants in  $\text{LiTi}_2(\text{PO}_4)_3$ . The Li-Frenkel (0.73 eV) is calculated to be the most favorable defect energy process ensuring the formation of Li vacancies required for the vacancy-assisted Li-ion migration. Long-range three-dimensional lithium vacancy migration was observed with a low activation energy of 0.36 eV, inferring fast Li-ion diffusion. The most favorable isovalent dopants on the Li and Ti sites are Na and Si, respectively. Li interstitials' formation in these materials is favored by doping of Ga on the Ti site. This engineering strategy can be of interest to improve the capacity of  $\text{LiTi}_2(\text{PO}_4)_3$ .

**Keywords:**  $\text{LiTi}_2(\text{PO}_4)_3$ ; electrolyte; defects; diffusion; dopants.

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## 1. Introduction

Phosphate based materials are of great interest in the development of rechargeable Li (or Na) ion batteries, catalysts, and optical devices due to their structural stability provided by phosphate  $(\text{PO}_4)^{3-}$  tetrahedral units [1-4]. Commercial Li-ion battery technology has been using olivine  $\text{LiFePO}_4$  as a conventional cathode material for the last thirty years [2,5-8]. A considerable experimental and theoretical research activity has been devoted to developing olivine  $\text{LiFePO}_4$  and other phosphate-based materials to prepare electrode materials for rechargeable Li-ion batteries [8,9-12].

$\text{LiTi}_2(\text{PO}_4)_3$  (LTP) has been proposed as a candidate electrolyte material in the application of Li-ion batteries due to its high Li-ion conductivity [13]. Electrochemical conductivity experiments together with  $^7\text{Li}$  NMR spectroscopy show that  $\text{Li}^+$  conduction is high in the as-prepared LTP [13]. A molecular dynamics simulation study carried out by Nuspl shows that the activation energy of lithium-ion migration is  $28.95 \text{ kJmol}^{-1}$  (0.30 eV), inferring fast ion conduction [14]. A first-principles study was applied to look at the diffusion properties of LTP and its derivatives. It was concluded that the activation energy of Li-ion diffusion is 0.41 eV and the substitution of Ti atoms leads to structural changes and diffusion of Li ions [15]. The effect of Ga substitution on the Ti site was studied by Liang *et al.* [16], and improved conductivity was noted compared with pure LTP. Though a few studies on the diffusion of  $\text{Li}^+$  ions and the electrochemical studies are available, experimental or theoretical reports on the intrinsic defects and solution of dopants are not available in the literature. While intrinsic defects influence the electrochemical behavior of a material, the thermal, mechanical, and electrical properties are dominated by dopants.

In this study, material modeling based on the classical pair- potentials was used to examine the intrinsic defects, diffusion of Li ions, and dopants' solution in LTP. This technique has been successfully applied to various ionic oxide materials in previous theoretical studies, including Li-ion battery and solid oxide fuel cell materials [8,9,17-21].

## 2. Computational Methods

All calculations were performed using a classical pair-wise potential simulation code GULP (General Utility Lattice Program) [22]. Interactions between ions were modeled using long-range (Coulombic) and short-range (Pauli repulsion and van der Waals attraction) forces. The Buckingham potentials (Table 1)[23-26] were used to describe Short-range repulsive forces. Structural relaxations were carried out using the Broyden-Fletcher-Goldfarb-Shanno (BFGS) algorithm [27]. In all relaxed configurations, forces on all atoms were smaller than 0.001 eV/Å. The Mott-Littleton method was used to model point defects and migrating ions [28]. Lithium-ion migration was calculated by considering seven interstitial points with equal intervals between neighbor lithium sites. Defect energies of migrating ions at seven points along the diffusion path were calculated. The mid-point between two adjacent O vacancy sites was used as the defect calculation center to reduce the systematic errors. The energy difference between the maximum local energy associated with the saddle point along this diffusion path and the lowest Li vacancy formation energy is calculated and reported as activation energy. In this method, ions are treated as spherical shapes with full charge at the dilute limit. Therefore, it is expected that defect energies will be overestimated. However, the trend in relative energies will be consistent [29].

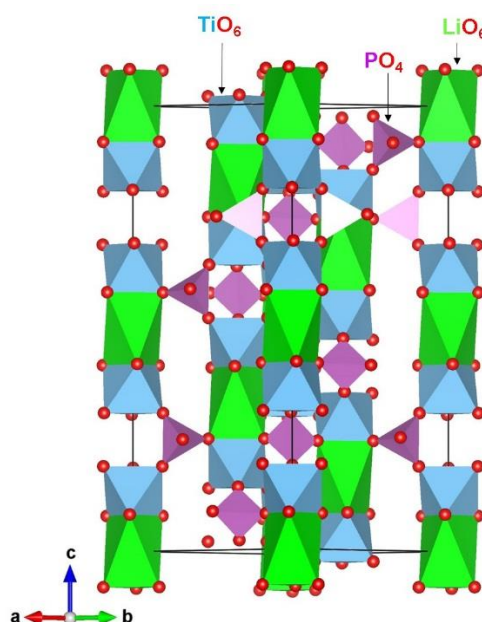
## 3. Results and Discussion

### 3.1. Crystal structure of $\text{LiTi}_2(\text{PO}_4)_3$ .

The crystal structure of LTP exhibits a trigonal crystallographic structure with space group  $R\bar{3}c$  (lattice parameters  $a = b = 8.5173 \text{ Å}$ ,  $c = 20.8595 \text{ Å}$ ,  $\alpha = \beta = 90.0^\circ$ ,  $\gamma = 120^\circ$ ) as reported by Redhammer *et al.* [30]. Figure 1 shows the crystal structure and P's chemical environment (forming a tetrahedral unit with adjacent four oxygen atoms) and Ti (forming an octahedral unit with adjacent six oxygen atoms). The quality of the Buckingham potentials (Table 1)[23-26] used in this study was validated by performing a geometry optimization calculation on the crystal structure of LTP. An excellent agreement between the calculated and experimental values was observed, indicating the efficacy of the potential parameters (Table 2).

**Table 1.** Buckingham potential parameters [23-26] used in the classical simulations of LTP. Two-body  $[\Phi_{ij}(r_{ij}) = A_{ij} \exp(-r_{ij}/\rho_{ij}) - C_{ij}/r_{ij}^6]$ , where A,  $\rho$ , and C are parameters which were selected carefully to reproduce the experimental data.

Interaction	$A / \text{eV}$	$\rho / \text{Å}$	$C / \text{eV} \cdot \text{Å}^6$	$Y / \text{e}$	$K / \text{eV} \cdot \text{Å}^{-2}$
$\text{Li}^+ - \text{O}^{2-}$	632.1018	0.2906	0.00	1.00	99999
$\text{Ti}^{4+} - \text{O}^{2-}$	5111.7	0.2625	0.00	-0.10	314.0
$\text{P}^{5+} - \text{O}^{2-}$	1273.42017	0.32272	0.000	5.00	99999
$\text{O}^{2-} - \text{O}^{2-}$	22764.00	0.1490	20.37	-2.00	15.52



**Figure 1.** Crystal structure of rhombohedral LTP.

**Table 2.** Comparison between calculated and experimental lattice parameters of LTP.

Parameter	Calculated	Experiment [30]	$ \Delta (\%)$
$a = b$ (Å)	8.4938	8.5173	0.28
$c$ (Å)	21.1528	20.8595	1.41
$\alpha = \beta$ (°)	90.0	90.0	0.00
$\gamma$ (°)	120.0	120.0	0.00
$V$ (Å <sup>3</sup> )	1321.60	1310.50	0.85

### 3.2. Intrinsic defects.

The electrochemical properties of a material can be influenced by intrinsic defects. Thus, a series of point defects (vacancies and interstitials) were considered. Then they were combined to calculate Schottky and Frenkel energies. Anti-site defect in which Li and Ti exchange their positions were also considered. We describe the Schottky, Frenkel, and anti-site defects by the following defect reaction equations using Kröger-Vink notation [31].

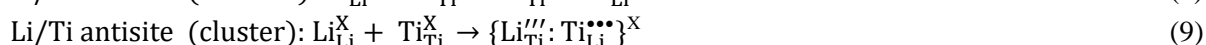
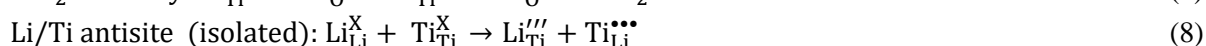
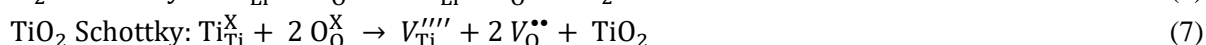
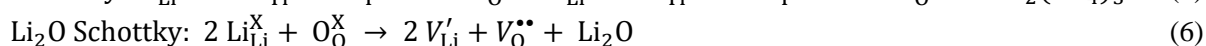
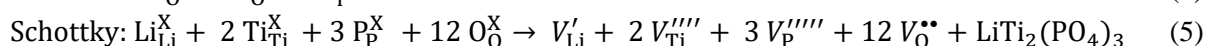
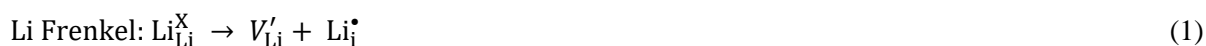
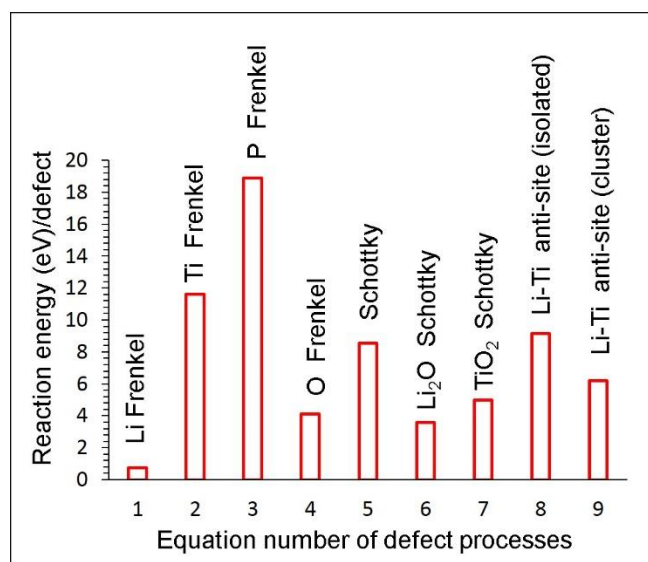


Figure 2 reports the defect reaction energies. The Li Frenkel is the lowest energy defect process with the defect energy of 0.73 eV, ensuring the formation of Li vacancies required for the vacancy mediated Li-ion diffusion. Furthermore, low Frenkel energies will ensure the formation of a high concentration of vacancies and interstitials, leading to the loss of crystal structure. The second most favorable defect is Li<sub>2</sub>O Schottky. However, the vacancies' concentration arising from these defects is not significant as this process is endothermic by

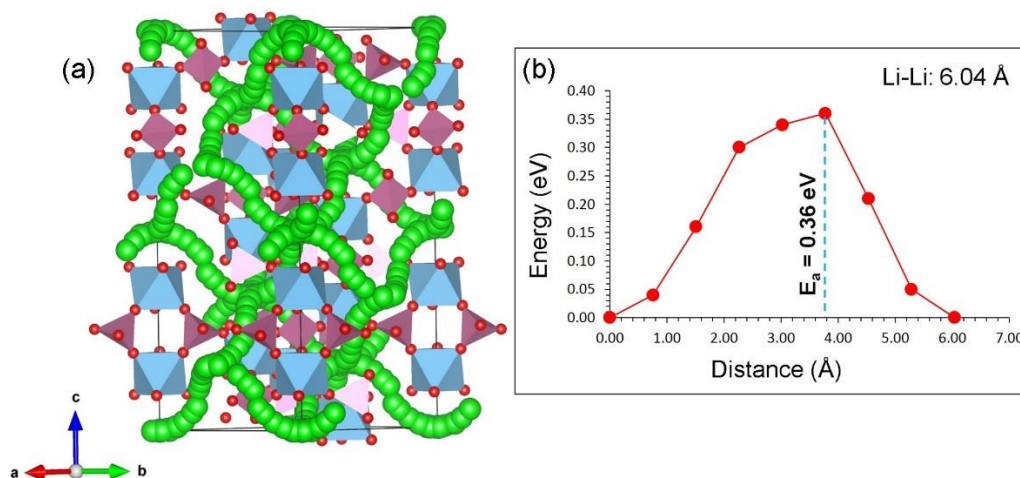
3.59 eV. Other Frenkel and Schottky defects are highly endoergic, meaning that they are not significant at room temperature. The Li-Ti anti-site defect cluster energy is calculated to be 6.21 eV. In this defect process, both  $\text{Li}^+$  and  $\text{Ti}^{4+}$  ions will exchange their positions simultaneously. Its isolated form exhibits high defect energy of 9.16 eV. High anti-site defect energies are due to the charge mismatch between  $\text{Li}^+$  and  $\text{Ti}^{4+}$ . The energy difference between these two forms of anti-site defect energies is the binding energy ( $-2.95$  eV), inferring the unstable nature of isolates defects ( $\text{Li}_{\text{Ti}}''' + \text{Ti}_{\text{Li}}^{4\bullet}$ ) and the preference of forming defect cluster  $\{\text{Li}_{\text{Ti}}''':\text{Ti}_{\text{Li}}^{4\bullet}\}^X$ . Other defect energies are highly endothermic, meaning that they are thermodynamically unfavorable.



**Figure 2.** Defect energies for different defect processes.

### 3.3. Diffusion of Li-ions.

To design an efficient battery, an electrolyte material with high ionic conductivity and low activation energy is necessary. As the diffusion of Li-ions in this material can be of interest in Li-ion batteries' application, we calculated the Li-ion diffusion pathways together with activation energies.



**Figure 3.** (a) Long-range Li vacancy migration path considered using local Li hops; (b) energy profile diagram for a local Li hop with its activation energy.

In general, the experimental investigation of diffusion pathways is challenging. The classical pair of potential based simulations can be used to calculate diffusion pathways and their

activation energies. In previous simulation studies [8,9, 32-35], a variety of ionic materials have been considered to calculate diffusion pathways together with activation energies. For example, a simulation study based on the classical pair potentials by Fisher *et al.* [9] reproduced the experimentally determined Li-ion diffusion pathway in LiFePO<sub>4</sub> [36].

A possible local Li hop with a jump distance of 6.04 Å was identified. Its activation energy was calculated to be 0.36 eV (Figure 3). This shows that the diffusion of Li-ions in this material is fast. A three-dimensional long-range diffusion pathway was constructed using this local hop, as shown in Figure 3a. The activation energy (0.36 eV) calculated in this study (Figure 3b) is in good agreement with the experimental value of 0.36 eV [16] and other theoretical values of 0.30 eV [14] and 0.41 eV [15].

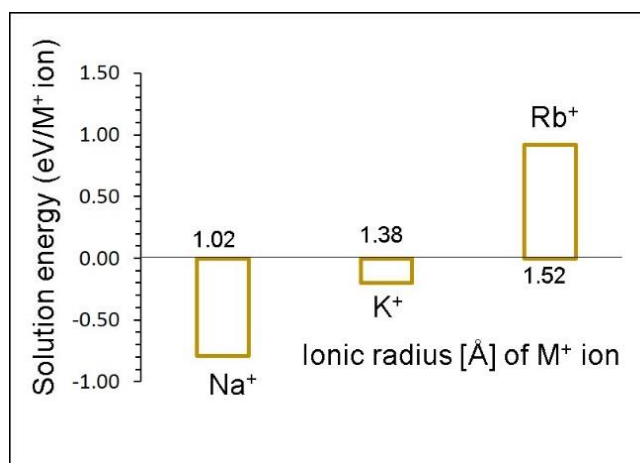
### 3.4. Solution of dopants.

Doping of appropriate dopants with different size or charge compared to the host atoms can tailor the properties of a material. Here we consider various isovalent and aliovalent dopants to screen and predict the promising dopants that can be considered for experiments. Solution energies were calculated using appropriate charge-compensating defects and lattice energies. Buckingham potentials used for dopants in this study are reported in the supplementary information (refer to Table S1).

Some monovalent dopants (M = Na, K, and Rb) on the Li site were first considered. Solution energies were calculated using the following reaction equation.



Exoergic solution energies were calculated for Na and K. The most favorable dopant is the Na with exothermic solution energy of −0.79 eV, suggesting that the synthesis of Li<sub>1-x</sub>Na<sub>x</sub>Ti<sub>2</sub>(PO<sub>4</sub>)<sub>3</sub> is possible. Exothermic solution energy for K (−0.20 eV) indicates that the experimental preparation of K-doped LTP is also worth trying. Doping of Rb exhibits endothermic solution energy, meaning that it is an unfavorable dopant (refer to Figure 4). Solution energy increases with the dopant size.



**Figure 4.** Solution energy of M<sub>2</sub>O (R=Na, K, and Rb) with respect to the M<sup>+</sup> ionic radius in LTP.

A range of trivalent dopants (Al, Co, Ga, Mn, Sc, In, Yb, Y, and Gd) was considered on the Ti site to introduce Li interstitials in the lattice. The capacity of LTP can be increased by doping of trivalent dopants on the Ti site. This dopant process will lead to the formation of Li interstitials as described by the following equation.

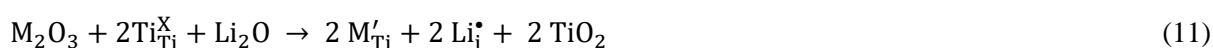
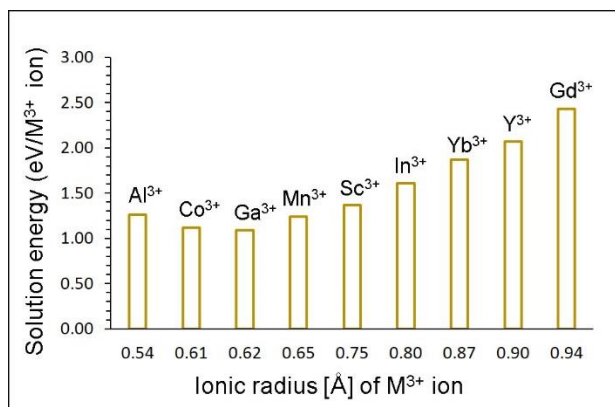


Figure 5 reports the solution energies. The most favorable dopant is Ga. Gallium doped LTO has been successfully synthesized by Liang *et al.* [16], and the improvement in the Li-ion conductivity was reported. Solution energies of Al, Co, and Mn are close to that of Ga, suggesting that these dopants are also worth testing experimentally. Solution energy increases with the increase of ionic radius from Ga to Gd. The highest solution energy is calculated for Gd, indicating that this dopant is highly unfavorable.



**Figure 5.** Solution energy of  $M_2O_3$  ( $M = \text{Al, Co, Ga, Mn, Sc, In, Yb, Y, and Gd}$ ) calculated for the formation of Li interstitial.

Finally, tetravalent dopants ( $M = \text{Si, Ge, Sn, Zr, and Ce}$ ) were considered at the Ti site. The following equation describes this doping process in which no charge compensation is necessary.



Solution energies are reported in Figure 6. The promising dopant for this process is found to be the Si with the exoergic solution energy of  $-1.55$  eV. The second most favorable dopant is the Ge with the solution energy of  $0.81$  eV. Solution energy increases with the ionic radius. High endothermic solution energies are noted for the other dopants, meaning they are unlikely to dope at normal temperatures.

#### 4. Conclusions

In this study, an atomistic simulation study based on the classical pair potentials was used to examine the intrinsic defects, diffusion of Li-ions, the dopant in  $\text{LiTi}_2(\text{PO}_4)_3$ . The Li Frenkel is the lowest energy defect process indicating that both Li vacancies and Li interstitials will be predominant at equilibrium conditions. The low activation energy of  $0.36$  eV shows that the Li-ion diffusion in these materials is high. It is found that  $\text{Na}^+$  and  $\text{Si}^{4+}$  are the prominent isovalent dopants on the Li and Ti sites, respectively. Li interstitials' formation can be facilitated in this material by doping of Ga on the Ti site.

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## Conflicts of Interest

The authors declare no conflict of interest.

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