

Ohmic and Viscous Dissipation Effect on Free and Forced Convective Flow of Casson Fluid in a Channel

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Received: 6.03.2021; Revised: 2.04.2021; Accepted: 5.04.2021; Published: 19.04.2021

Abstract: Analytical study of the free and forced convective flow of Casson fluid in the existence of viscous dissipation, ohmic effect and uniform magnetic field in a porous channel to the physical model. The nonlinear coupled partial differential equations are converted to linear partial differential equations using similarity transformation and the classical perturbation method. The physical parameters such as Prandtl number (Pr), viscous dissipation (Vi), Schmidt number (Sc), Reynolds number (R), thermal buoyancy parameter (λ), Ohmic number (Oh), Casson fluid parameter (β), Darcy number (Da), Hartmann number (M₂), the concentration of buoyancy parameter (N), chemical reaction rate (γ) effect on velocity, temperature and concentration have been studied with pictorial representation. For the particular case, the present paper analysis is compared with the previous work and is found good agreement.

Keywords: Casson fluid, perturbation technique, magnetic field, porous medium, ohmic effect, viscous dissipation.

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1. Introduction

Among non-Newtonian fluids, Casson fluid has gained the attention of researchers because of its wide applications in the field of engineering such as food processing, metallurgy, drilling and bioengineering operations have been mentioned by Ramesh *et al.* [1]. The free and forced convective flows in a vertical channel to study the effect of Joule heating and viscous dissipation have been studied both analytically and numerically by using the power series method by Barletta *et al.* [2]. The steady laminar MHD mixed convection of viscous dissipating fluid about a permeable vertical fluid to study the ohmic effect and viscous dissipation in the presence of the magnetic field, to solve the equations Keller box method is implemented, temperature and velocity profiles are studied for the physical model by Orhan Aydın *et al.* [3]. The effect of ohmic heating and viscous dissipation effect on an incompressible, viscous, steady flow of an electrically conducting fluid in the presence of a magnetic field at a stagnation point has been studied by Pushkar Raj Sharma *et al.* [4]. The governing equations are solved numerically by using the fourth-order Runge-Kutta method with the shooting technique. Temperature and velocity profiles are discussed graphically.

Das *et al.* [5] investigated the Joule heating and viscous dissipation effect on thermal and momentum transport of MHD flow in the presence of opposing and aiding buoyancy force over the inclined plate. Similarity transformation and Runge–Kutta fourth-order method-based shooting method has been implemented to solve the governing equations numerically and results on temperature, velocity, heat transfer rate, and shear stress have been discussed. A steady MHD forced convective flow of viscous fluid in the presence of Joule heating and viscous dissipation in a porous medium have been studied and distribution of velocity and temperature distributions have been discussed by Raju *et al.* [6]. Gilbert Makanda [7] have studied the radiation effect on MHD free convection of Casson fluid in the presence of the non-Darcy porous medium. The obtained boundary layer equations are converted into non-similar partial differential equations and are solved by using the bivariate quasi linearization method and temperature and velocity profiles at boundary level are discussed.

An unsteady natural convective flow on the plate in the existence of the radiation, porous medium and chemical reaction to study the effect of ohmic heating and viscous dissipation have been studied numerically by Lognathan *et al.* [8] in which the obtained equations are solved by Crank Nicolson method of finite difference scheme. Physical parameters are discussed on velocity, concentration, skin friction, temperature and Nusselt number distributions. A double-diffusive flow above the heated vertical plate in the existence of different properties of the fluid to study the combined effect of internal heat generation and viscous dissipation have been carried by Suresh Babu *et al.* [9] numerically using shooting technique and the effect of physical parameters are carried out on velocity, temperature, concentration graphically. Suresh Babu *et al.* [10] studied the viscous dissipation and ohmic effect on the oscillatory flow of a couple of stress fluid on a semi-infinite vertical permeable plate in a porous medium.

The boundary layer steady flow and heat transfer over a porous plate in presence of viscous dissipation, thermal radiation and magnetic field have been studied numerically using shooting technique by Ravi Kumar *et al.* [11]. They have stated that greater viscous dissipative heat causes an increase in the temperature of the fluid. Raju *et al.* [12] have studied the convective condition and cross-diffusion of Casson fluid across a paraboloid of revolution. The shooting technique is used to solve the governing equations and the effects are studied on velocity, concentration and temperature. The study of MHD viscous flow over the slandering stretching surface in the presence of Joule, Ohmic and Forchheimer effect have been studied by Divya *et al.* [13]. MHD Casson fluid flows on a stretched surface with variable thickness to study the thermal radiation effect has been investigated by Basavaraju *et al.* [14].

Ohmic and magnetic effects on an unsteady stretching surface to study the heat transfer have been discussed by Divya *et al.* [15]. The homotopy perturbation method is used to convert the nonlinear partial differential equations into ODE. Taseer *et al.* [16] investigated the stretched 3D flow of viscous fluid with prescribed concentration and heat fluxes in the existence of Joule heating, chemical reaction and viscous dissipation. Ganesh *et al.* [17] studied the viscous dissipation and Joule heating of Oldroyd B fluid in three dimensions. The governing equations have been solved using the shooting method and discussed the graphs about the viscous dissipation and Joule heating effect.

Ashish *et al.* [18] investigated the influence of ohmic effect and viscous dissipation, heat absorption, or generation on MHD flow of nanofluid over the stretching sheet with injection and suction. The governing equations have been solved numerically using the shooting technique and the results have been discussed. Anjalidevi *et al.* [19] considered the unsteady boundary layer flow of hydrodynamic fluid on stretching surfaces to study the effect of viscous dissipation, mass

transfer, thermal radiation and ohmic dissipation. The governing equations have been solved numerically by using the fourth-order Range-Kutta method and the effects of physical parameters have been discussed. Hitesh Kumar [20] studied the characteristics of heat transfer and flow of the fluid in the presence of ohmic heating, viscous dissipation and radiation on a stretching porous sheet. The obtained governing equations have been solved analytically by the Homotopy perturbation method and the results have been discussed on temperature and velocity profiles.

Dulal [21] investigated the unsteady electrically conducting a convective flow of the fluid on a permeable stretching sheet in the presence of an Ohmic effect, viscous dissipation and internal heat. The effect of physical parameters such that buoyancy parameter, Eckert number, unsteadiness parameters have been discussed. Thiagarajan *et al.* [22] studied the double stratified magnetohydrodynamic nanofluid flow on an exponentially stretching sheet in the existence of heat absorption or generation under the influence of Joules heat and viscous dissipation. Nachtsheim-Swigert shooting technique scheme with fourth-order Runge-Kutta method has been employed to solve the governing equations. Further, the effect of physical parameters has been discussed on temperature and concentration.

The effect of viscous dissipation, heat and mass transfer on mixed convection of MHD flow of Casson fluid over a plate has been investigated by Gireesha *et al.* [23], in which governing equations were solved using RK4-5 scheme and convergent solutions for velocity, concentration and temperature fields were discussed graphically. A steady, incompressible, electrically conducting mixed convective MHD flow of viscous fluid over an infinite plate in the presence of ohmic heating, chemical reaction and viscous dissipation have been studied by Garg *et al.* [24] in which the effect of non-dimensionalized parameters on velocity, temperature and concentration profiles have been studied. MHD mixed convection, incompressible and electrically conducting nanofluid flow along with the vertical plate with ohmic and viscous dissipation have studied numerically using Quasi-linearization technique and finite difference scheme by Jagadha *et al.* [25]. The mixed convective flows of Casson fluid in a porous channel in presence of amplification and porous media have been studied by Shilpa *et al.* [26]. Perturbation technique is implemented to solve the obtained governing equations and physical parameters effects have been studied on temperature, velocity and concentration. Sravan Kumara *et al.* [27] investigated the unsteady convection flow of nanofluid on exponentially moving vertical plates in the presence of viscous dissipation and Lorentz force.

Wilfred *et al.* [28] studied the effect of ohmic and viscous dissipation on convective of a MHD flow over the shrinking surface in the existence of radiation and internal heat generation. Umamaheswar *et al.* [29] discussed the effect of ohmic and viscous dissipation of magnetohydrodynamic flow with suction and injection in the presence of porous media. The effect of physical parameters has been discussed on temperature, concentration and velocity. Manjula *et al.* [30] examined the free convection, steady MHD of Casson fluid flow over a slanted porous sheet in the presence of radiation and viscous dissipation. Runge-Kutta method of fourth-order has been implemented to solve the dimensionless equations. Further temperature, velocity and concentration profiles have been discussed. Tariq Javed *et al.* [31] investigated the effect of ohmic heating and viscous dissipation by considering the Prandtl nanofluid with a magnetic coating on an unsteady moveable surface. The Keller-Box method has been implemented to solve the governing equations and physical parameters have been discussed.

In all the above-cited literature and to our knowledge, it has been observed that the numerical approach carries out the study of the casson fluid flow and much work has not been carried out by incorporating ohmic and viscous dissipation effects, which play a great role in

controlling the velocity, concentration and temperature for many applications cited above. Thus, the main objective and aim of the present work are to study the viscous dissipation and ohmic effect on forced and free convection flow of Casson fluid in a porous channel in the presence of uniform magnetic field porous medium and chemical reaction analytically by using perturbation technique. The effects of non-dimensional parameters on temperature, velocity and concentration have been studied graphically.

2. Materials and Methods

A steady, laminar, incompressible Casson fluid flow in the presence of viscous dissipation, ohmic effect and applied magnetic field B_0 uniformly in a porous channel is the physical configuration to be considered. This model consists of a two-dimensional system of axisymmetric flow with one wall of the channel at $y = -H$ and another wall at $y = +H$. The x -axis is along with the channel's flow, parallel to the channel surface and perpendicular to the channel will be the y -axis. At $y = +H$ the fluid is injected into the channel and at $y = -H$ the fluid is extracted out from the channel with uniform velocity V .

The governing equations for MHD flow of Casson fluid in the presence of amplification, porous media, viscous dissipation, and Ohmic effect are expressed as follows

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left(1 + \frac{1}{\beta}\right) \frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2 u}{\rho} - \frac{\mu_p}{k_2} u \pm g(\beta_T(T-T_2) + \beta_c(C-C_2)), \quad (2)$$

$$u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial y} - \frac{\mu_p}{k} v + \nu \left(1 + \frac{1}{\beta}\right) \frac{\partial^2 v}{\partial x^2}, \quad (3)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k}{\rho c_p} \frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\rho c_p} \left(\frac{\partial u}{\partial y}\right)^2 + \frac{\mu_p}{\rho c_p k_2} u^2, \quad (4)$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} - Ck_1. \quad (5)$$

Where σ is electrical conductivity, ρ is density, k is thermal conductivity, c_p is the specific heat at constant pressure, β is Casson fluid parameter, ν is kinematic viscosity, T is the temperature of the fluid, k_1 is reaction rate, C is the concentration field, μ is dynamic viscosity, k_2 is the permeability of the medium and D is mass diffusion.

For the above mentioned physical configuration, the boundary conditions are:

$$u = 0, v = \frac{V}{2}, T = T_2, C = C_2 \text{ at } y=H, \\ \frac{\partial u}{\partial y} = 0, v = 0, T = T_1, C = C_1 \text{ at } y=0. \quad (6)$$

From (3) and (4), we get

$$u \frac{\partial^2 u}{\partial x \partial y} + \frac{\partial u}{\partial x} \frac{\partial u}{\partial y} + v \frac{\partial^2 u}{\partial y^2} + \frac{\partial v}{\partial y} \frac{\partial u}{\partial y} - u \frac{\partial^2 v}{\partial x^2} - \frac{\partial u}{\partial x} \frac{\partial v}{\partial x} - \frac{\partial v}{\partial x} \frac{\partial v}{\partial y} - v \frac{\partial^2 v}{\partial x \partial y} = \nu \left(1 + \frac{1}{\beta}\right) \left(\frac{\partial^3 u}{\partial y^3} - \frac{\partial^3 v}{\partial x^3}\right) - \left(\frac{\sigma B_0^2}{\rho} + \frac{\mu_p}{k_2}\right) \frac{\partial u}{\partial y} + \frac{\mu_p}{k_2} \frac{\partial v}{\partial x} \pm \frac{\partial g}{\partial y} [\beta_T(T - T_2) + \beta_c(C - C_2)] \quad (7)$$

The above equations are non-dimensionalized by introducing the following parameters

$$y^* = \frac{y}{H}, \quad x^* = \frac{x}{H}, \quad v = aVf(y^*), \quad u = -Vx^*f'(y^*), \quad \phi(y^*) = \frac{C-C_2}{C_1-C_2}, \quad \theta(y^*) = \frac{T-T_2}{T_1-T_2}, \quad (8)$$

With these parameters (4), (5) and (7) becomes,

$$\theta'' - aP_r f \epsilon \theta' + \epsilon Vi f'^2 + Oh \epsilon f^2 = 0, \quad (9)$$

$$\phi'' - a \varepsilon \text{Sc} f \phi' - \text{Sc} \gamma (\phi + A) = 0. \quad \text{Where } A = \frac{c_2}{c_1 - c_2} \quad (10)$$

$$\left(1 + \frac{1}{\beta}\right) f'''' - (M^2 + \text{Da}) f'' + \text{Re}[-a f f'' + (2 - a) f' f''] \pm \lambda [N \phi' + \theta'] = 0, \quad (11)$$

The reduced boundary conditions are

$$f(0) = 0, f(1) = \frac{1}{2}, f''(0) = 0, \phi(0) = 1, f'(1) = 0, \theta(0) = 1, \phi(1) = 0, \theta(1) = 0, \quad (12)$$

Where $\text{Oh} = \frac{(aVH)^2}{K2^2(T1-T2)}$ is Ohmic effect, $\text{Gr}_x = \frac{VH^4 g \beta_T (T1-T2)}{x v^3}$ is Grashof number, $M^2 = \frac{\sigma B_0^2 H^2}{\mu}$ is Hartmann number, $R = \frac{VH}{v}$ is Reynolds Number ($R < 0$ for injection and $R > 0$ for suction), $\text{Vi} = \frac{\mu a^2 v^2}{k2(T1-T2)}$ is viscous dissipation, $N = \frac{\beta_C(C1-C2)}{\beta_T(T1-T2)}$ is a concentration of buoyancy parameter, $\text{Sc} = \frac{HV}{D}$ is Schmidt number, $\gamma = \frac{k_1 H}{v}$ is chemical reaction rate, $\text{Pr} = \frac{\rho C_p HV}{k}$ is Prandtl number, $\text{Da} = \frac{H^2 \mu_p}{K2v}$ is Darcy number and $\lambda = \frac{\text{Gr}_x}{R^2}$ is the thermal buoyancy parameter.

2.1. Method of solution.

To find the analytic solution of (9), (10) and (11), we consider the solution by using the classical perturbation technique for velocity f , temperature θ , concentration ϕ as given below with very small values of perturbation parameter ' ε '.

$$\begin{aligned} \theta &= \theta_0 + \varepsilon * \theta_1, \\ \phi &= \phi_0 + \varepsilon * \phi_1, \\ f &= f_0 + \varepsilon * f_1, \end{aligned} \quad (13)$$

On solving (9), (10), and (11) by perturbation technique using (13), we get

$$\theta_0'' = 0,$$

$$\theta'' - a \text{Pr} f_0 \theta' + \text{Vi} f_0'^2 + \text{Oh} f_0'^2 = 0, \quad (14)$$

$$\phi_0'' - \text{Sc} \gamma \phi_0 - \text{Sc} \gamma A = 0,$$

$$\phi_1'' - a \text{Sc} f_0 \phi_0' - \text{Sc} \gamma \phi_1 = 0. \quad (15)$$

$$\left(1 + \frac{1}{\beta}\right) f_0'''' - (M^2 + \text{Da}) f_0'' \pm \lambda [\theta_0' + N \phi_0'] = 0,$$

$$\left(1 + \frac{1}{\beta}\right) f_1'''' - (M^2 + \text{Da}) f_1'' + R[(2 - a) f_0' f_0'' - a f_0 f_0'''] \pm \lambda [\theta_1' + N \phi_1'] = 0, \quad (16)$$

The corresponding boundary condition takes the form

$$f_1(1) = 0, f_0(1) = \frac{1}{2}, f_0'(1) = 0, f_1'(1) = 0, \theta_0(1) = 0, \theta_1(1) = 0, \phi_0(1) = 0, \phi_1(1) = 0, \\ f_0''(0) = 0, f_1''(0) = 0, f_0(0) = 0, f_1(0) = 0, \theta_0(0) = 1, \theta_1(0) = 0, \phi_0(0) = 1, \phi_1(0) = 0.$$

Solutions (14), (15) and (16) under the above boundary conditions are

$$f = d_9 + d_{10}y + d_{11}e^{d_8y} + d_{12}e^{-d_8y} + d_{13}y^2 + d_{14}e^{d_{11}y} + d_{15}e^{-d_{11}y} + \varepsilon(d_{126} + d_{127}y + d_{128}e^{d_8y} + d_{129}e^{-d_8y} + d_{231}y^2 + d_{232}y^3 + d_{233}y^4 + d_{234}y^5 + d_{165}y^6 + d_{168}y^7 + d_{235}e^{d_{11}y} + d_{236}e^{-d_{11}y} + d_{237}ye^{d_8y} + d_{239}y^2e^{d_8y} + d_{238}ye^{-d_8y} + d_{240}y^2e^{-d_8y} + d_{241}ye^{d_{11}y} + d_{242}ye^{-d_{11}y} + d_{243}y^2e^{d_{11}y} + d_{244}y^2e^{-d_{11}y} + d_{194}y^3e^{d_8y} + d_{198}y^3e^{-d_8y} + d_{209}e^{2d_8y} + d_{210}e^{-2d_8y} + d_{211}e^{2d_{11}y} + d_{212}e^{-2d_{11}y} + d_{213}e^{d_{18}y} + d_{214}e^{d_{19}y} + d_{215}e^{d_{20}y} + d_{216}e^{d_{21}y} + d_{220}y^3e^{d_{11}y} + d_{227}y^3e^{-d_{11}y}),$$

$$\theta = d_4 + d_5y + \varepsilon(d_{47} + d_{48}y + d_{73}y^2 + d_{74}y^3 + d_{110}e^{d_8y} + d_{111}e^{-d_8y} + d_{77}y^4 + d_{78}y^5 + d_{112}e^{d_{11}y} + d_{113}e^{-d_{11}y} + d_{114}ye^{d_8y} + d_{115}ye^{-d_8y} + d_{116}ye^{d_{11}y} + d_{117}ye^{-d_{11}y} + d_{85}y^2e^{d_8y} + d_{88}y^2e^{-d_8y} + d_{95}y^2e^{d_{11}y} + d_{98}y^2e^{-d_{11}y} + d_{101}e^{2d_8y} + d_{102}e^{-2d_8y} + d_{103}e^{2d_{11}y} + d_{104}e^{-2d_{11}y} + d_{105}e^{d_{18}y} + d_{106}e^{d_{19}y} + d_{107}e^{d_{20}y} + d_{108}e^{d_{21}y} + d_{109}y^6),$$

$$\phi = d_2 e^{d_1 y} + d_3 e^{-d_1 y} - A + \varepsilon (d_{16} e^{d_1 y} + d_{17} e^{-d_1 y} + d_{42} y e^{d_1 y} + d_{43} y e^{-d_1 y} + d_{44} y^2 e^{d_1 y} + d_{45} y^2 e^{-d_1 y} + d_{46} + d_{28} e^{d_{18} y} + d_{29} e^{d_{19} y} + d_{30} e^{d_{20} y} + d_{31} e^{d_{21} y} + d_{32} e^{2d_1 y} + d_{35} e^{-2d_1 y} + d_{36} y^3 e^{d_1 y} + d_{39} y^3 e^{-d_1 y}).$$

All the d_i terms which are mentioned in the above solutions are constants and their expressions are mentioned in the appendix.

3. Result and Discussion

Here, in this problem, an analytical approach is carried out to convert the coupled PDE to a simultaneous ODE using the similarity transformation and perturbation method with its parameter ' ε '. By this technique, the physical model's governing differential equations are transferred to a six set of analytical expressions of ODE are obtained with its complete solutions under the boundary conditions. Mathematica software is used for the computational work to study the physical parameters such as Prandtl number, viscous dissipation, Schmidt number, thermal buoyancy parameter, ohmic number, Reynolds number, Casson fluid parameter, Darcy number, Hartmann number, the concentration of buoyancy parameter, chemical reaction rate on temperature, velocity and concentration.

3.1. Effect of Casson parameter (β).

Casson fluid is one of the non-Newtonian fluid which has high viscosity at zero rate shear. Fig.1 and Fig.2 indicate the Casson fluid parameter (β) effect on concentration, temperature, and velocity. The concentration, velocity, and temperature profiles diminish with the increase in the Casson parameter. This is due to the non-Newtonian character of the Casson fluid, which offers more resistance flow.

3.2. Effect of Darcy number (Da).

Fig.3 shows the effect of Darcy number (Da) on concentration and temperature as the porous medium's permeability increases the concentration and temperature profiles, enhancing with a very small increase in Darcy number. Here, we are highlighting more on concentration and thermal effect, which is not focused in the literature.

3.3. Effect of buoyancy parameter (λ).

Fig. 4 shows the Buoyancy parameter on concentration and temperature in which Buoyancy parameter (λ) enhances the concentration and temperature profiles, Due to the thickness of the fluid velocity profiles are not showing much effect.

3.4. Effect of Hartmann number (M^2).

Fig.5 and Fig.6 indicate the Hartmann number effect (M^2) on concentration, temperature and velocity. In all the cases, the profiles decrease with the increasing value of Hartmann number due to the magnetic field present in the electrically conducting fluid, which offers resisting force to flow called Lorentz's force, which acts against the flow.

3.5. *Effect of Concentration of buoyancy parameter (N).*

The concentration of buoyancy parameter increases the concentration and temperature profiles at the lower values of N increases. This is due to the increased thickness of the fluid, and no change in velocity is seen in Fig.7.

3.6. *Effect of chemical reaction (γ).*

As the chemical reaction occurs, there will be an exchange of molecules in the fluid so that the concentration and temperature profiles increasing with increasing values of chemical reaction (γ) are seen in Fig.8.

3.7. *Effect of ohmic number (Oh).*

The ohmic effect arises in the fluid due to the porosity of the media. Fig.9 and Fig.10 show the ohmic effect on concentration, temperature, and velocity. The profiles enhance with the increase in ohmic number (Oh) due to the porous media.

3.8. *Effect of Prandtl number (Pr).*

Fig.11 and Fig.12 show the effect of the Prandtl number on concentration, temperature and velocity. All the profiles diminish as the Prandtl number (Pr) increases due to the high viscosity of the fluid.

3.9. *Effect of Reynolds number (R).*

Reynolds number is the ratio of inertial force to viscous force. As the Reynolds number increases, inertial forces increase and viscosity decreases. Fig.13 indicates the effect of Reynolds Number (R) on concentration and temperature. Viscosity decreases with an increase in the inertial forces, which enhances the profiles of concentration and temperature.

3.10. *Effect of Schmidt's number (Sc).*

Schmidt's number is the ratio of momentum diffusivity to mass diffusivity. Schmidt's number (Sc) effect on concentration and temperature is seen in Fig.14, in which the profiles increase with an increase in very small values of Schmidts number.

3.11. *Effect of viscous dissipation (Vi).*

Fig.15 gives the effect of viscous dissipation (Vi) on concentration and temperature. As the viscous dissipation increases, the fluid's thickness enhances, leading to increased concentration and temperature profiles and no changes in velocity due to an increase in thickness of the fluid.

3.12. *Effect of perturbation parameter (ϵ) and amplification parameter (a).*

We have solved the problem in the presence of amplification by using the perturbation method. Fig.16 and Fig.17 indicates the effect of perturbation parameter (ϵ) and amplification parameter (a) on concentration and temperature, both concentration and temperature profiles increases with an increase in the values of the parameters mentioned above, so these two parameters are used as a tool to control concentration and temperature, but these parameters are not showing the much effect on velocity.

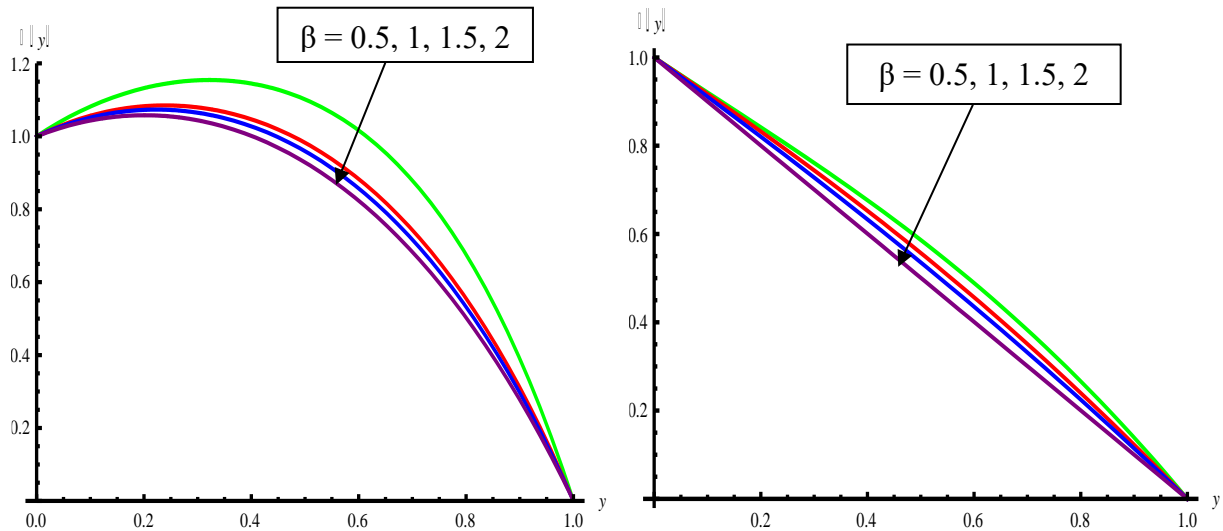


Figure 1. Casson Parameter (β) effect on Concentration and Temperature.

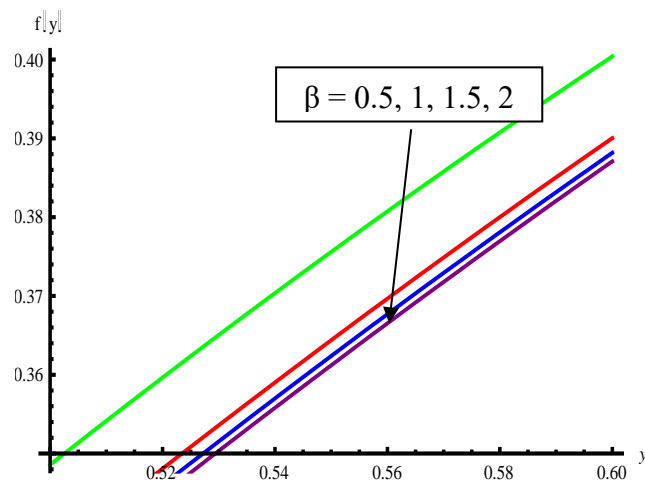


Figure 2. Casson Parameter (β) effect on velocity.

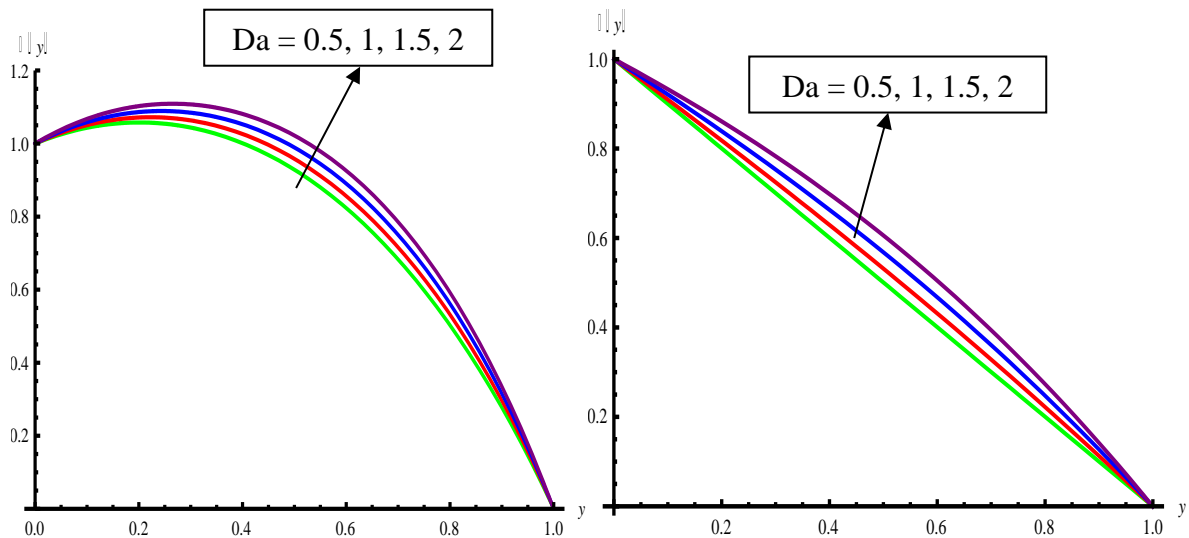


Figure 3. Darcy Number (Da) effect on Concentration and Temperature.

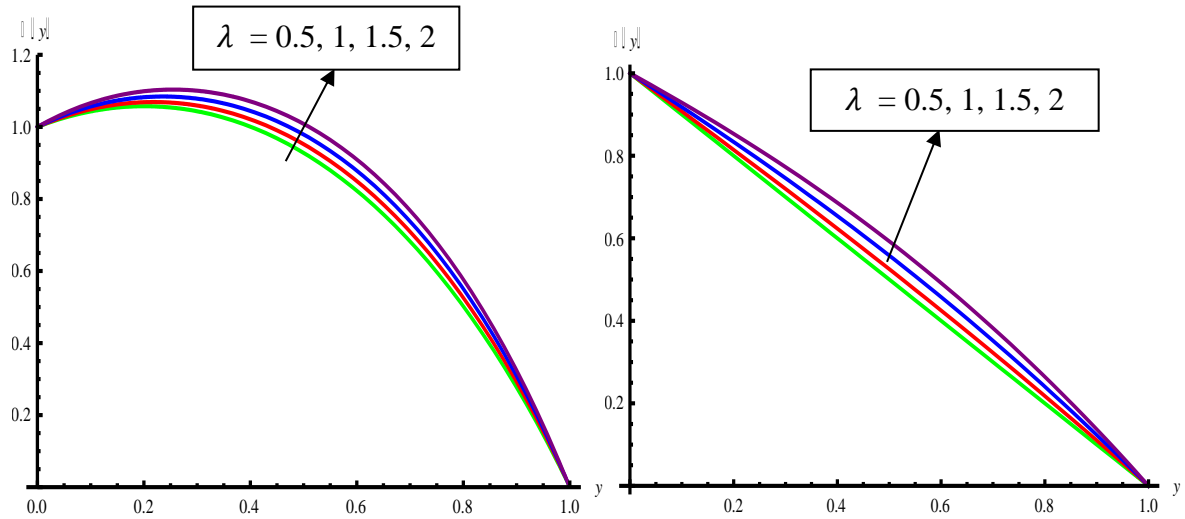


Figure 4. Buoyancy Parameter (λ) effect on Concentration and Temperature.

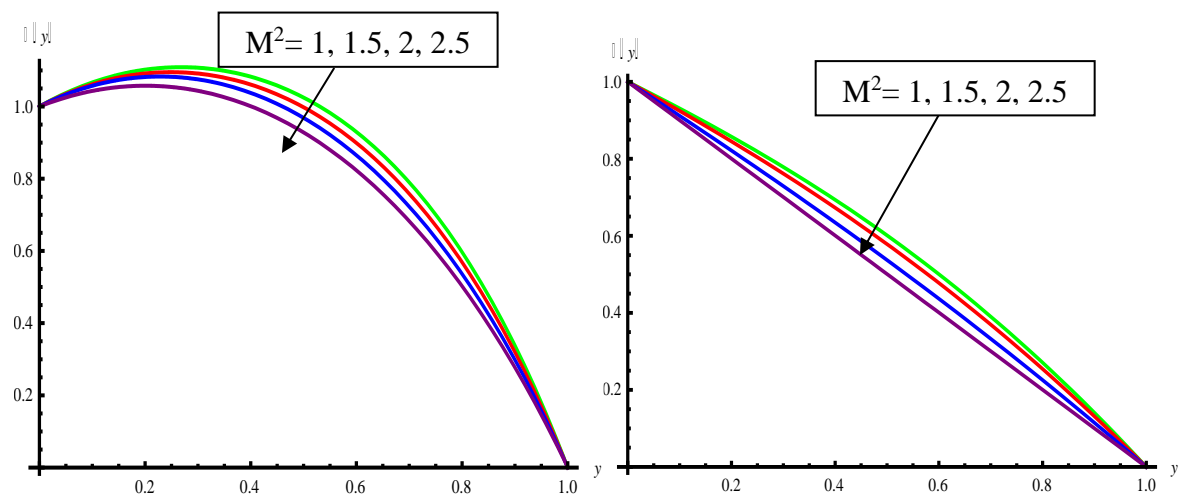


Figure 5. Hartmann number (M^2) effect on Concentration and Temperature.

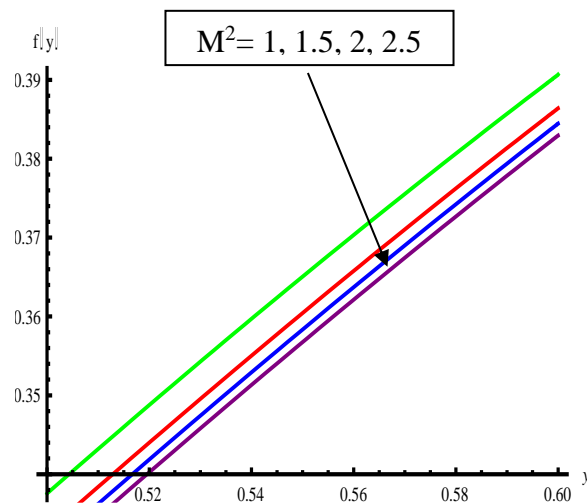


Figure 6. Hartmann number (M^2) effect on velocity.

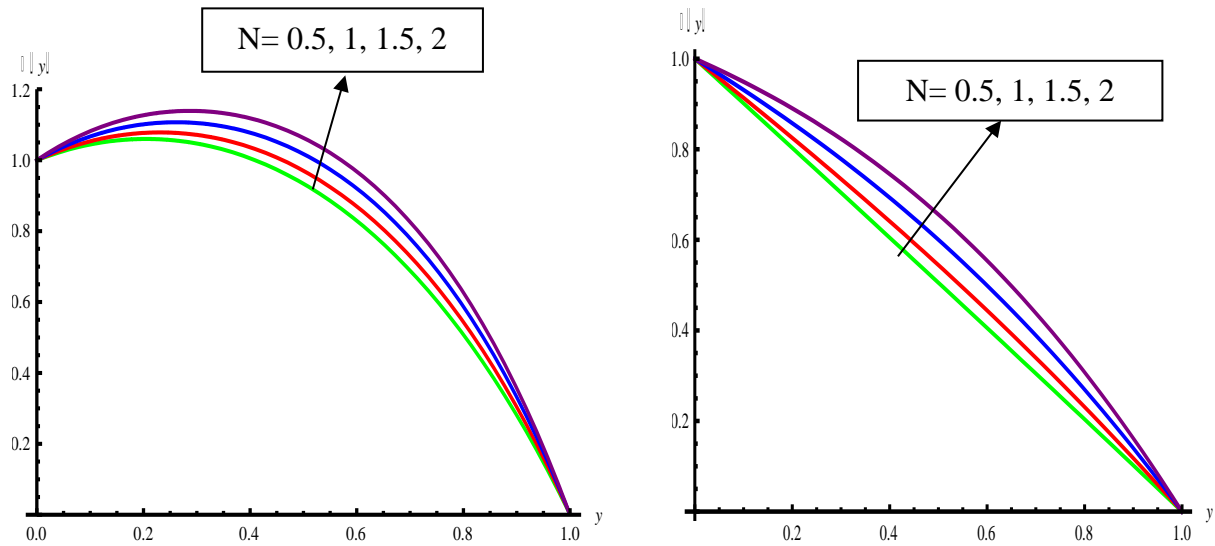


Figure 7. Concentration Buoyancy Parameter (N) effect on Concentration and Temperature.

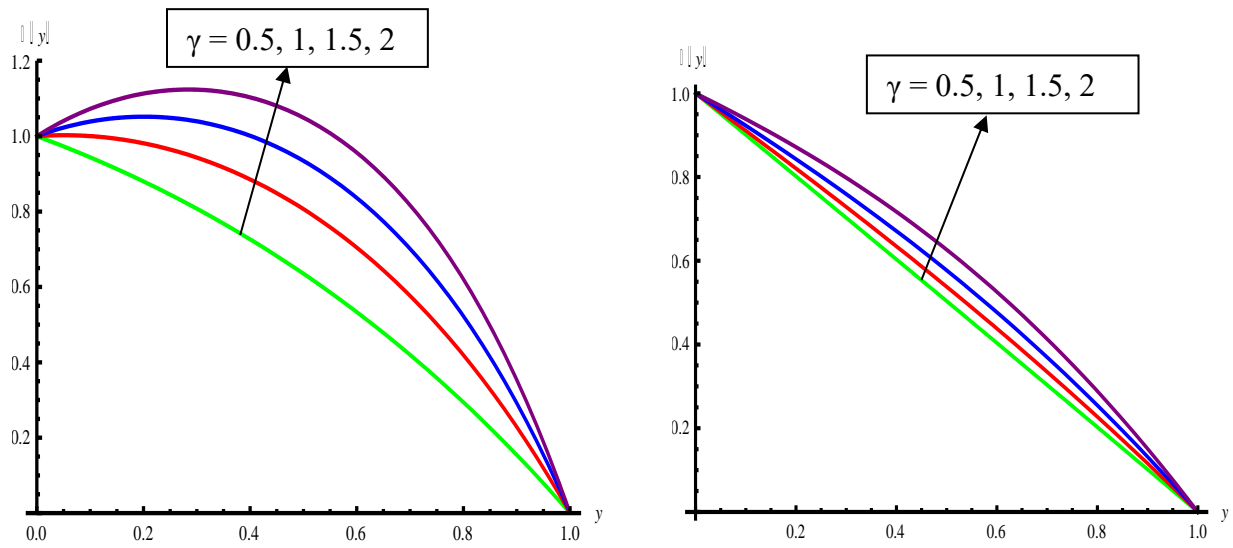


Figure 8. Chemical Reaction (γ) effect on Concentration and Temperature.

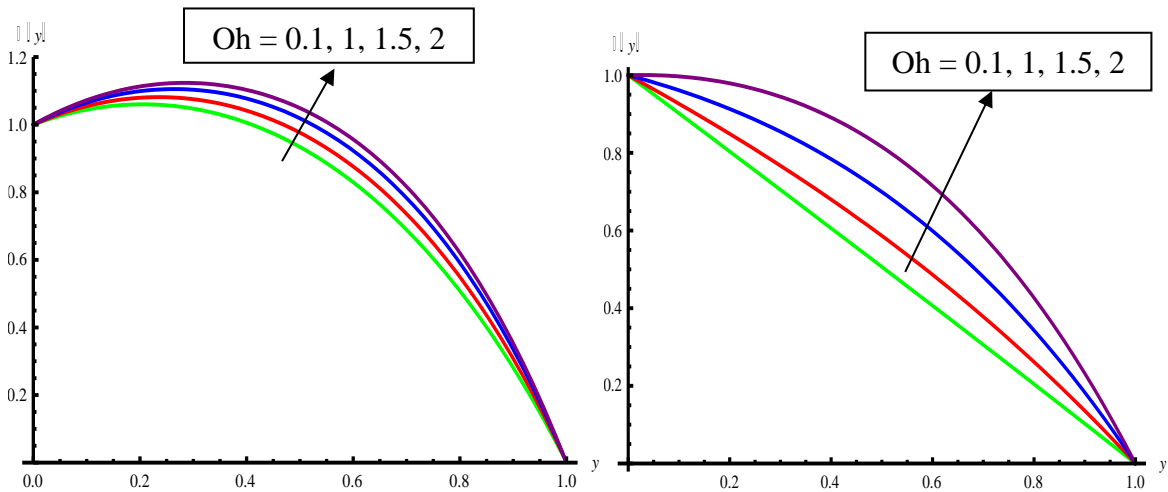


Figure 9. Ohmic number (Oh) effect on Concentration and Temperature.

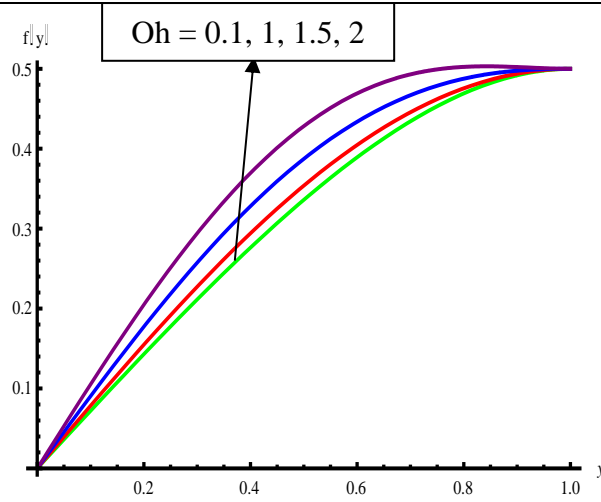


Figure 10. Effect of Ohmic number (Oh) on velocity.

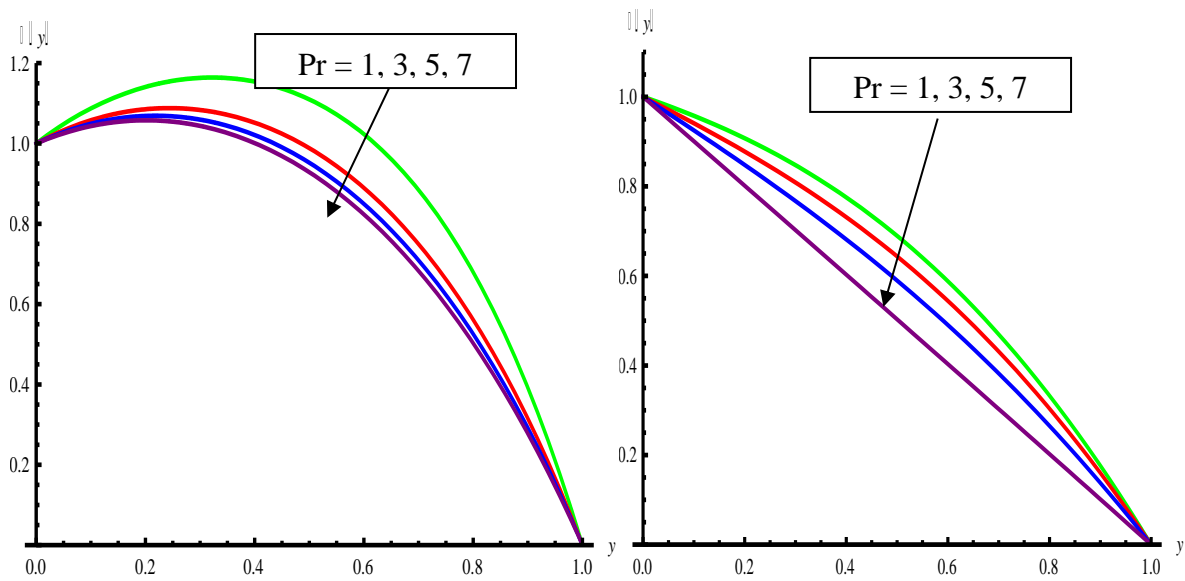


Figure 11. Prandtl number (Pr) effect on concentration and temperature.

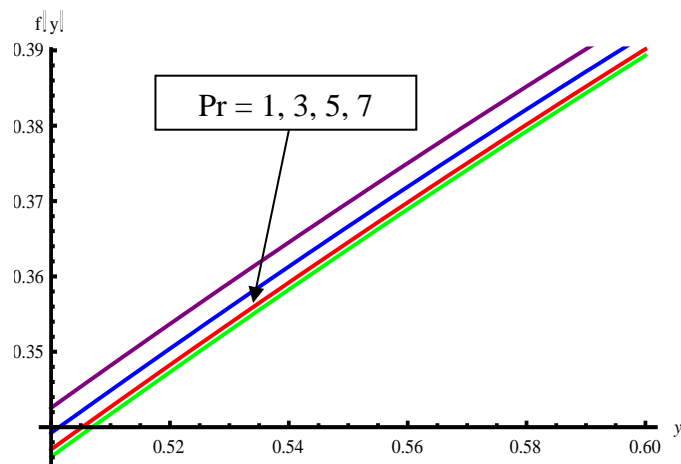


Figure 12. Prandtl number (Pr) effect on velocity.

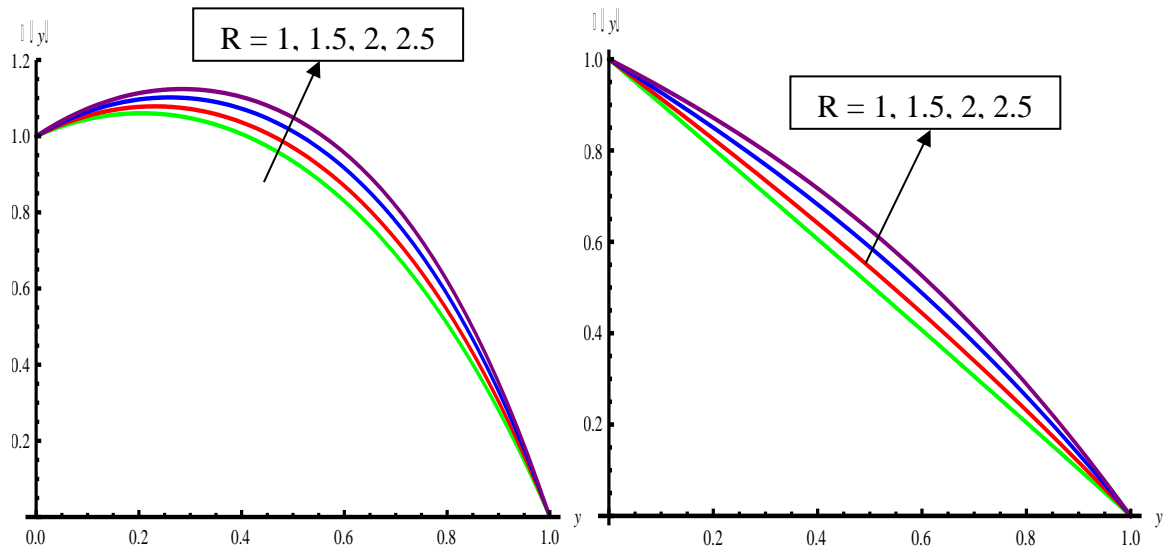


Figure 13. Reynolds number (R) effect on concentration and temperature.

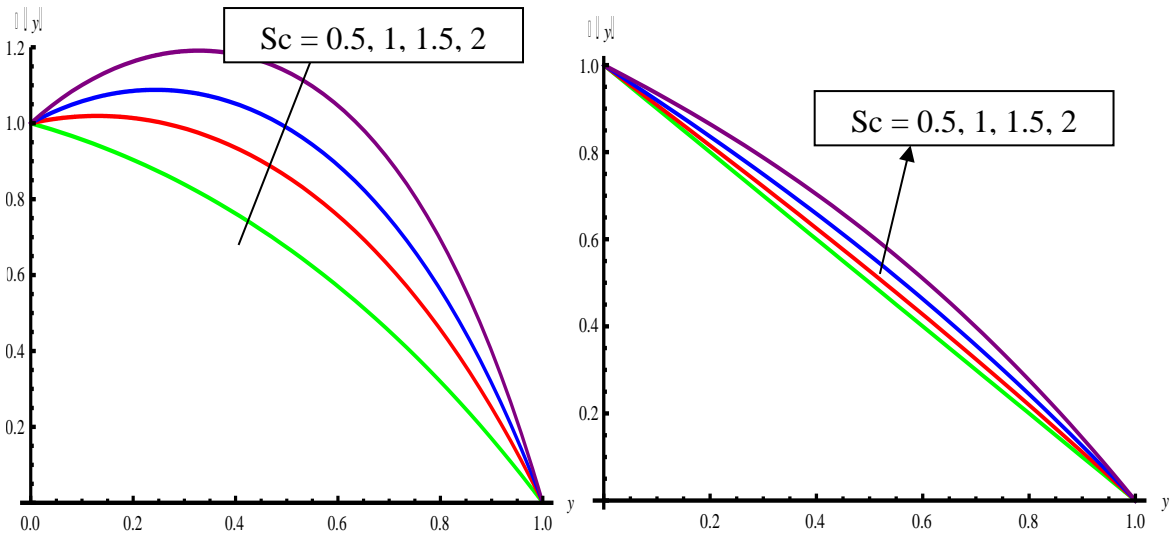


Figure 14. Schmidt's number (Sc) effect on concentration and temperature.

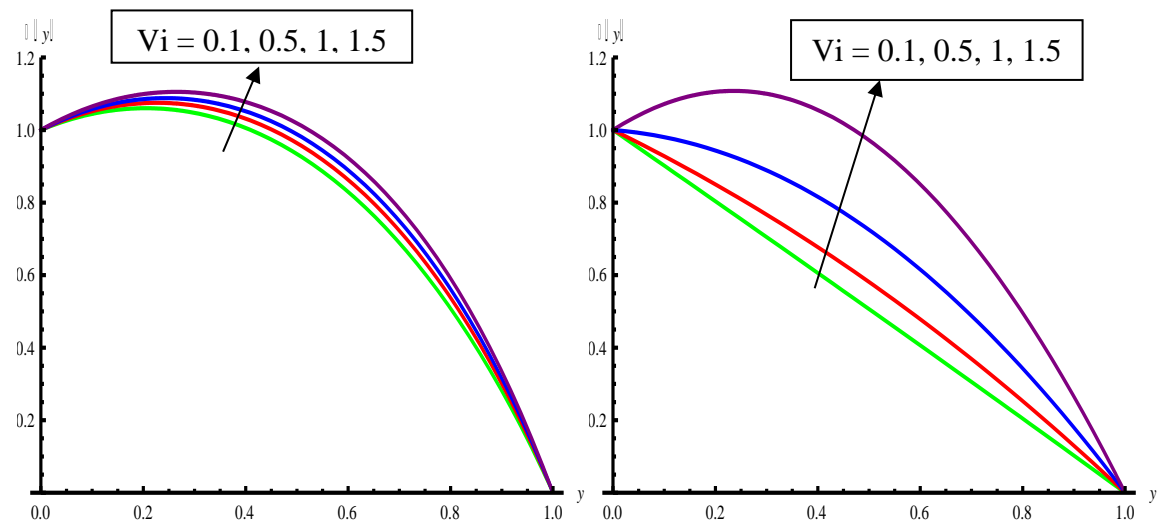


Figure 15. Viscous dissipation (Vi) effect on concentration and temperature.

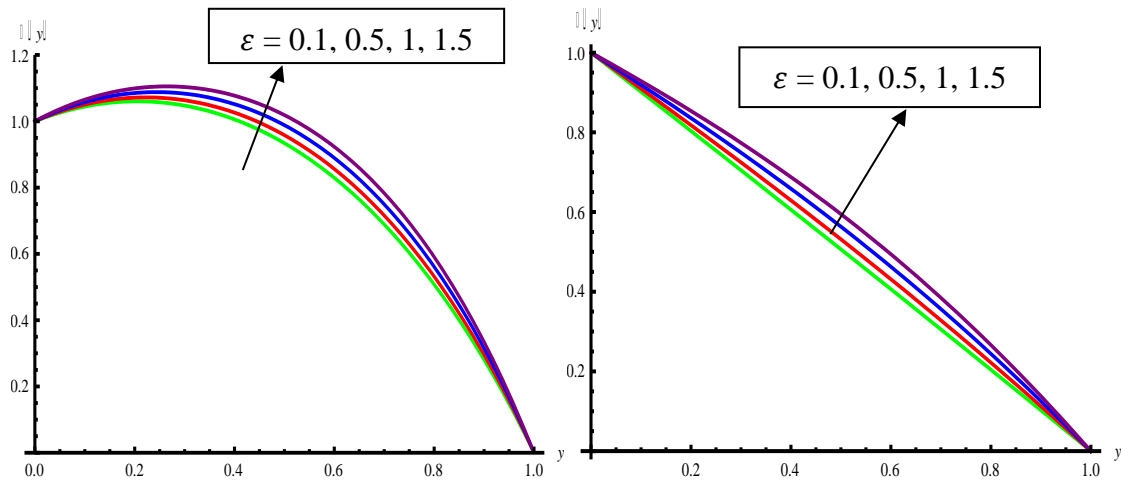


Figure 16. Effect of perturbation parameter (ε) on concentration and temperature.

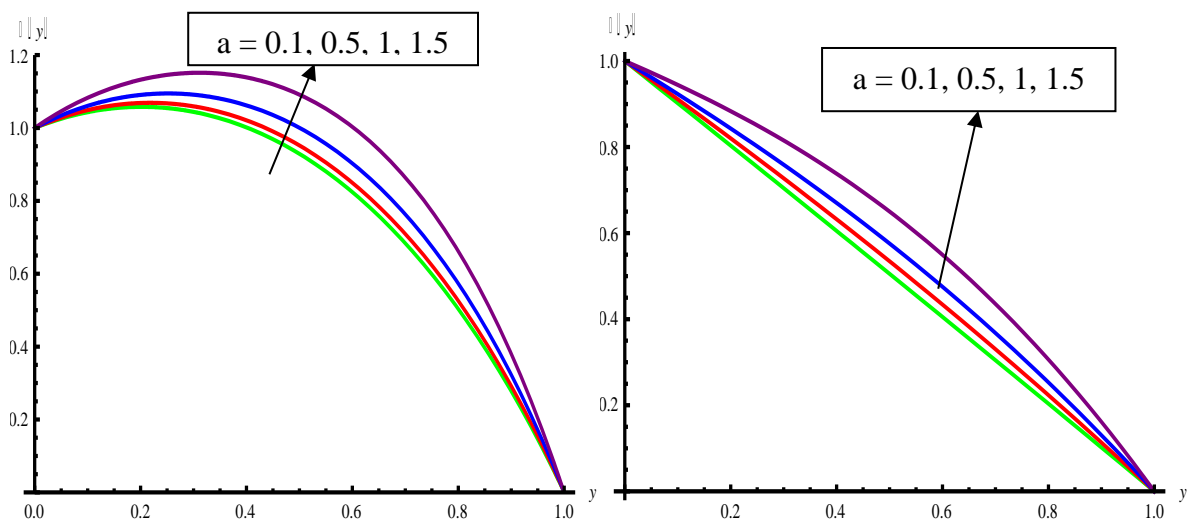


Figure 17. Amplification parameter (a) effect on concentration and temperature.

4. Conclusions

Due to the above assumptions, the physical parameters show more effect on concentration and temperature than velocity. Temperature, concentration, and velocity profile decrease with Casson's increase because of the Casson fluid's non-Newtonian character. Darcy number, Buoyancy parameter, Concentration buoyancy parameters enhance concentration and temperature profiles. Hartmann number and Prandtl number lowers the profiles because of the presence of magnetic field and high viscosity, respectively. The ohmic effect on temperature, concentration and velocity profiles enhances with an increase in ohmic number, whereas temperature and concentration profiles enhance with increases in viscous dissipation. Schmidt's Number, Reynold's number, amplification and perturbation parameters enhance the concentration, temperature profiles. Our results coincide with the earlier work carried out by Shilpa *et al.* [26] in the absence of ohmic effect, viscous dissipation and porous medium.

Funding

This research received no external funding.

Acknowledgments

The authors of the paper would like to sincerely thank the M S Ramaiah Institute of Technology and Vivekananda Institute of Technology, affiliated with VTU, to collaborate towards the research work.

Conflicts of Interest

The authors declare no conflict of interest.

Appendix

$$\begin{aligned}
 d_1 &= \sqrt{sc * nu}; A = \frac{c2}{c1-c2}; d_2 = \frac{A*(1-e^{-d1})-e^{-d1}}{(e^{d1}-e^{-d1})}; d_3 = \frac{-A*(1-e^{d1})+e^{d1}}{(e^{d1}-e^{-d1})}; d_4 = 1; d_5 = -1; d_6 = 1 + \frac{1}{\beta}; \\
 d_7 &= m^2 + Da; d_{13} = \frac{\lambda d_5}{-2d_7}; d_{14} = \frac{\lambda N d_1 d_2}{d_6 d_1^4 - d_7 d_1^2}; d_{15} = \frac{-\lambda N d_1 d_3}{d_6 d_1^4 - d_7 d_1^2}; d_{18} = d_1 + d_8; d_{19} = d_8 - d_1; d_{20} = \\
 d_1 - d_8; d_{21} &= d_1 + d_8; d_{22} = \frac{asc d_1 d_2 d_9}{2d_1}; d_{23} = \frac{asc d_1 d_3 d_9}{2d_1}; d_{24} = \frac{asc d_1 d_2 d_{10}}{4d_1}; d_{25} = \frac{-asc d_1 d_2 d_{10}}{4d_1^2}; d_{26} = \\
 \frac{asc d_1 d_3 d_{10}}{4d_1}; d_{27} &= \frac{asc d_1 d_3 d_{10}}{4d_1^2}; d_{28} = \frac{asc d_1 d_2 d_{11}}{d_{18}^2 - sc * nu}; d_{29} = \frac{-asc d_1 d_3 d_{11}}{d_{19}^2 - sc * nu}; d_{30} = \frac{asc d_1 d_2 d_{12}}{d_{20}^2 - sc * nu}; d_{31} = \frac{-asc d_1 d_3 d_{12}}{d_{21}^2 - sc * nu}; \\
 d_{32} &= \frac{asc d_1 d_2 d_{14}}{4d_1^2 - sc * nu}; d_{33} = \frac{asc d_1 d_2 d_{14}}{sc * nu}; d_{34} = \frac{asc d_1 d_2 d_{15}}{-sc * nu}; d_{35} = \frac{-asc d_1 d_3 d_{15}}{4d_1^2 - sc * nu}; d_{36} = \frac{asc d_1 d_2 d_{13}}{6d_1}; d_{37} = \\
 \frac{-asc d_1 d_2 d_{13}}{4d_1^2}; d_{38} &= \frac{asc d_1 d_2 d_{13}}{4d_1^3}; d_{39} = \frac{asc d_1 d_3 d_{13}}{6d_1}; d_{40} = \frac{asc d_1 d_3 d_{13}}{4d_1^2}; d_{41} = \frac{asc d_1 d_3 d_{13}}{4d_1^3}; d_{42} = d_{22} + d_{25} + \\
 d_{38}; d_{43} &= d_{23} + d_{27} + d_{41}; d_{44} = d_{24} + d_{37}; d_{45} = d_{26} + d_{40}; d_{46} = d_{33} + d_{34}; d_{49} = [ap_r d_5 d_9 - \\
 O(d_9 + 2d_{11}d_{12} + 2d_{14}d_{15}) - V(d_{10}^2 - 2d_8^2 d_{11}d_{12} - 2d_1^2 d_{14}d_{15})]; d_{50} &= ap_r d_5 d_{10} - 2 * Od_9 d_{10} - \\
 4Vd_{10}d_{13}; d_{51} &= ap_r d_5 d_{11} - 2 * Od_9 d_{11} - 2Vd_{10}d_8 d_{11}; d_{52} = ap_r d_5 d_{12} - 2 * Od_9 d_{12} + 2 * \\
 Vd_{10}d_8 d_{12}; d_{53} &= ap_r d_5 d_{13} - Od_9 d_1 - Od_{10}^2 - Od_9 d_{13} - V4d_{13}^2; d_{54} = -2 * Od_{10}d_{13}; d_{55} = \\
 ap_r d_5 d_{14} - 2 * Od_9 d_{14} - 2Vd_{10}d_{14}; d_{56} &= ap_r d_5 d_{15} - 2 * Od_9 d_{15} + 2Vd_{10}d_{15}; d_{57} = -2 * \\
 Od_{10}d_{11} - V4d_8 d_{11}d_{13}; d_{58} &= -2 * Od_{10}d_{12} + V4d_8 d_{12}d_{13}; d_{59} = -2 * Od_{11}d_{13}; d_{60} = -2d_{12}d_{13}; \\
 d_{61} &= -2 * Od_{10}d_{14} - 4vd_1 d_{13}d_{14}; d_{62} = -2 * Od_{10}d_{15} + 4vd_1 d_{13}d_{15}; d_{63} = -2 * Od_{13}d_{14}; d_{64} = \\
 -O * 2d_{13}d_{15}; d_{65} &= -Od_{11}^2 - Vd_8^2 d_{11}^2; d_{66} = -O * d_{12}^2 - Vd_8^2 d_{12}^2; d_{67} = -O * d_{14}^2 - Vd_1^2 d_{14}^2; d_{68} = \\
 -O * d_{15}^2 - Vd_1^2 d_{15}^2; d_{69} &= -O * 2d_{11}d_{14} - 2Vd_1 d_8 d_{11}d_{14}; d_{70} = -O * 2d_{11}d_{15} + 2 * Vd_8 d_{11}d_{15}; \\
 d_{71} &= -O * 2d_{12}d_{14} + Vd_8 d_{12}d_{14} * 2; d_{72} = -O * 2d_{12}d_{15} - 2Vd_1 d_8 d_{12}d_{15}; d_{73} = \frac{d_{49}}{2}; d_{74} = \frac{d_{50}}{6}; \\
 d_{75} &= \frac{d_{51}}{d_8^2}; d_{76} = \frac{d_{52}}{d_8^2}; d_{77} = \frac{d_{53}}{12}; d_{78} = \frac{d_{54}}{20}; d_{79} = \frac{d_{55}}{d_1^2}; d_{80} = \frac{d_{56}}{d_1^2}; d_{81} = \frac{d_{57}}{d_8^2}; d_{82} = -\frac{2d_{57}}{d_8^3}; d_{83} = \frac{d_{58}}{d_8^2}; \\
 d_{84} &= \frac{2d_{58}}{d_8^3}; d_{85} = \frac{d_{59}}{d_8^2}; d_{86} = \frac{-4d_{59}}{d_8^3}; d_{87} = \frac{6d_{59}}{d_8^4}; d_{88} = \frac{d_{60}}{d_8^2}; d_{89} = \frac{4d_{60}}{d_8^3}; d_{90} = \frac{6d_{60}}{d_8^4}; d_{91} = \frac{d_{61}}{d_1^2}; d_{92} = \\
 \frac{-2d_{61}}{d_1^3}; d_{93} &= \frac{d_{62}}{d_1^2}; d_{94} = \frac{2d_{62}}{d_1^3}; d_{95} = \frac{d_{63}}{d_1^2}; d_{96} = \frac{-4d_{63}}{d_1^3}; d_{97} = \frac{6d_{63}}{d_1^4}; d_{98} = \frac{d_{64}}{d_1^2}; d_{99} = \frac{4d_{64}}{d_1^3}; d_{100} = \frac{6d_{64}}{d_1^4}; \\
 d_{101} &= \frac{d_{65}}{4d_8^2}; d_{102} = \frac{d_{66}}{4d_8^2}; d_{103} = \frac{d_{67}}{4d_1^2}; d_{104} = \frac{d_{68}}{4d_1^2}; d_{105} = \frac{d_{69}}{d_{18}^2}; d_{106} = \frac{d_{70}}{d_{19}^2}; d_{107} = \frac{d_{71}}{d_{20}^2}; d_{108} = \frac{d_{72}}{d_{21}^2}; d_{109} = \\
 \frac{-O*d_{13}}{30}; d_{110} &= d_{75} + d_{87} + d_{82}; d_{111} = d_{76} + d_{84} + d_{90}; d_{112} = d_{79} + d_{92} + d_{97}; d_{113} = d_{80} + d_{94} + \\
 d_{100}; d_{114} &= d_{86} + d_{81}; d_{115} = d_{83} + d_{89}; d_{116} = d_{91} + d_{96}; d_{117} = d_{93} + d_{99}; d_{118} = d_{110}d_8 + d_{114}; \\
 d_{119} &= -d_8 d_{111} + d_{115}; d_{120} = d_{112}d_1 + d_{116}; d_{121} = -d_1 d_{113} + d_{117}; d_{122} = d_1 d_{116} + 2d_{95}; d_{123} = \\
 -d_1 d_{117} + 2d_{98}; d_{124} &= d_{114}d_8 + 2d_{85}; d_{125} = -d_8 d_{115} + 2d_{88}; d_{130} = -R(2 - a); d_{131} = \lambda d_{48} + \\
 2d_{130}d_{10}d_{13}; d_{132} &= 2\lambda d_{73} + 4d_{13}^2 d_{130}; d_{133} = \lambda d_{118} + d_{130}d_{10}d_{11}d_8^2 + 2d_{13}d_{130}d_8 d_{11} + Ra d_9 d_8^3 d_4; \\
 d_{134} &= \lambda d_{119} + d_{130}d_{10}d_8^2 d_{12} - 2d_{13}d_8 d_{12}d_{130} - Ra d_9 d_8^3 d_{12}; d_{135} = \lambda d_{120} + \lambda N d_1 d_{16} + \lambda N d_{42} + \\
 d_{130}d_{10}d_1^2 d_{14} + 2d_1 d_{13}d_{14}d_{130} + Ra d_9 d_1^3 d_{14}; d_{136} &= \lambda d_{121} + \lambda N d_{43} - \lambda N d_1 d_{17} + d_{130}d_{10}d_1^2 d_{15} - \\
 2d_1 d_{13}d_{15}d_{130} - Ra d_9 d_1^3 d_{15}; d_{137} &= \lambda d_{122} + \lambda N d_1 d_{42} + 2\lambda N d_{44} + 2d_{13}d_{130}d_1^2 d_{14} + Ra d_{10}d_1^3 d_{14}; \\
 d_{138} &= \lambda d_{123} - \lambda N d_1 d_{43} + 2\lambda N d_{45} + 2d_{13}d_{130}d_1^2 d_{15} - Ra d_{10}d_1^3 d_{15}; d_{139} = \lambda d_{124} + \\
 2d_{13}d_{130}d_{11}d_8^2 + Ra d_{10}d_8^3 d_{11}; d_{140} &= \lambda d_{125} + 2d_{13}d_{130}d_{12}d_8^2 - Ra d_{10}d_8^3 d_{12}; d_{141} = \lambda d_8 d_{85} + \\
 Ra d_{13}d_{11}d_8^3; d_{142} &= -\lambda d_8 d_{88} + Rad_{12} d_{13}d_8^3; d_{143} = \lambda d_1 d_{95} + \lambda N(d_1 d_{44} + 3d_{36}) + Rad_{13} d_{14}d_1^3;
 \end{aligned}$$

$$\begin{aligned}
 d_{144} &= -\lambda d_1 d_{98} + \lambda N(-d_1 d_{45} + 3d_{39}) - \text{Rad}_{13} d_1^3 d_{15}; & d_{145} &= 2\lambda d_8 d_{101} + d_{130} d_{11}^2 d_8^3 + \text{Rad}_{11}^2 d_8^3; \\
 d_{146} &= -2\lambda d_8 d_{102} + d_{130} d_{12}^2 d_8^3 + \text{Rad}_{12}^2 d_8^3; & d_{147} &= 2\lambda N d_1 d_{32} + 2\lambda d_1 d_{103} + \text{Rad}_1^3 d_{14}^2 + d_{130} d_1^3 d_{14}^2; \\
 d_{148} &= -2\lambda d_1 d_{104} - 2\lambda N d_1 d_{35} - d_{130} d_1^3 d_{15}^2 + \text{Rad}_1^3 d_{15}^2; & d_{149} &= \lambda N d_{18} d_{28} + \lambda d_{18} d_{105} + \\
 & d_{130} d_8 d_{11} d_1^2 d_{14} + d_{130} d_1 d_{14} d_8^2 d_{11} + \text{Rad}_{11} d_1^3 d_{14} + \text{Rad}_{14} d_{11} d_8^3; & d_{150} &= \lambda N d_{19} d_{29} + \lambda d_{19} d_{106} + \\
 & d_{130} d_8 d_{11} d_1^2 d_{15} - d_{130} d_1 d_{15} d_8^2 d_{11} - \text{Rad}_{11} d_1^3 d_{15} + \text{Rad}_{15} d_{11} d_8^3; & d_{151} &= \lambda N d_{20} d_{107} + \lambda N d_{20} d_{30} - \\
 & d_{130} d_8 d_{12} d_1^2 d_{14} + d_{130} d_1 d_{12} d_{14} d_8^2 + \text{Rad}_{12} d_1^3 d_{14} - \text{Rad}_{14} d_{12} d_8^3; & d_{152} &= \lambda N d_{21} d_{108} + \lambda d_{31} d_{21} - \\
 & d_{130} d_8 d_{12} d_1^2 d_{15} - d_{130} d_1 d_{15} d_8^2 d_{12} - \text{Rad}_{12} d_1^3 d_{15} - \text{Rad}_{15} d_{12} d_8^3; & d_{153} &= \lambda N d_1 d_{36}; d_{154} = -\lambda N d_1 d_{39}; \\
 d_{155} &= \frac{d_{131}}{-2d_7}; & d_{156} &= \frac{-d_{132}}{6d_7}; & d_{157} &= \frac{d_{133}}{4d_8^3 d_6 - d_7 d_{28}}; & d_{158} &= \frac{d_{134}}{-4d_8^3 d_6 + 2d_7 d_8}; & d_{159} &= \frac{d_{135}}{d_6 d_1^4 - d_7 d_1^2}; & d_{160} &= \\
 & \frac{d_{136}}{d_6 d_1^4 - d_7 d_1^2}; & d_{161} &= \frac{-3\lambda d_{74}}{12d_7}; & d_{162} &= \frac{-3\lambda d_6 d_{74}}{d_7^2}; & d_{163} &= \frac{-4\lambda d_{77}}{12d_7}; & d_{164} &= -\frac{4\lambda d_6 d_{77}}{d_7^2}; & d_{165} &= \frac{-5\lambda d_{78}}{30d_7}; & d_{166} &= \\
 & \frac{-5\lambda d_{78} d_6}{d_7^2}; & d_{167} &= \frac{-60\lambda d_{78} d_6^2}{d_7^3}; & d_{168} &= \frac{-6\lambda d_{109}}{42d_7}; & d_{169} &= \frac{-6\lambda d_{109} d_6}{d_7^2}; & d_{170} &= \frac{24\lambda d_{109} d_6^2}{d_7^3}; & d_{171} &= d_6 d_1^4 - d_7 d_1^2; \\
 d_{172} &= 4d_6 d_1^3 - 2d_1 d_7; & d_{173} &= 6d_6 d_1^2 - d_7; & d_{174} &= 4d_6 d_1; & d_{175} &= \frac{d_{137}}{d_{171}}; & d_{176} &= \frac{-d_{172} d_{137}}{d_{171}^2}; & d_{177} &= \\
 & d_6 d_1^4 - d_7 d_1^2; & d_{178} &= -4d_6 d_1^3 + 2d_1 d_7; & d_{179} &= 6d_6 d_1^2 - d_7; & d_{180} &= -4d_6 d_1; & d_{181} &= \frac{d_{138}}{d_{177}}; & d_{182} &= \\
 & \frac{-d_{178} d_{138}}{d_{177}^2}; & d_{183} &= 4d_6 d_8^3 - 2d_8 d_7; & d_{184} &= 6d_6 d_8^2 - d_7; & d_{185} &= 4d_6 d_1; & d_{186} &= \frac{d_{139}}{2*d_{183}}; & d_{187} &= \frac{-d_{139} d_{184}}{d_{183}^2}; \\
 d_{188} &= -4d_6 d_8^3 + 2d_8 d_7; & d_{189} &= 6d_6 d_8^2 - d_7; & d_{190} &= -4d_6 d_8; & d_{191} &= \frac{d_{140}}{2*d_{188}}; & d_{192} &= \frac{-d_{140} d_{189}}{d_{188}^2}; & d_{193} &= \\
 & 2d_{184}^2 - 2d_{183} d_{185}; & d_{194} &= \frac{d_{141}}{3d_{183}}; & d_{195} &= \frac{-d_{184} d_{141}}{d_{183}^2}; & d_{196} &= \frac{d_{141} d_{193}}{d_{183}^3}; & d_{197} &= 2d_{189}^2 - 2d_{190} d_{188}; & d_{198} &= \\
 & \frac{d_{142}}{3d_{188}}; & d_{199} &= \frac{d_{189} d_{142}}{d_{188}^2}; & d_{200} &= \frac{d_{142} d_{197}}{d_{188}^3}; & d_{201} &= 2d_{172}^2 - 2d_{173} d_{171}; & d_{202} &= \frac{d_{143}}{d_{171}}; & d_{203} &= \frac{-2d_{172} d_{143}}{d_{171}^2}; \\
 d_{204} &= \frac{d_{143} d_{201}}{d_{171}^3}; & d_{205} &= 2d_{178}^2 - 2d_{177} d_{179}; & d_{206} &= \frac{d_{144}}{d_{177}}; & d_{207} &= \frac{-2d_{178} d_{144}}{d_{177}^2}; & d_{208} &= \frac{d_{144} d_{205}}{d_{177}^3}; & d_{209} &= \\
 & \frac{d_{145}}{16d_6 d_8^4 - 4d_7 d_8^2}; & d_{210} &= \frac{d_{146}}{16d_6 d_8^4 - 4d_7 d_8^2}; & d_{211} &= \frac{d_{147}}{16d_6 d_1^4 - 4d_7 d_1^2}; & d_{212} &= \frac{d_{148}}{16d_6 d_1^4 - 4d_7 d_1^2}; & d_{213} &= \\
 & \frac{d_{149}}{d_6 d_{18}^4 - d_7 d_{18}^2}; & d_{214} &= \frac{d_{150}}{d_6 d_{19}^4 - d_7 d_{19}^2}; & d_{215} &= \frac{d_{151}}{d_6 d_{20}^4 - d_7 d_{20}^2}; & d_{216} &= \frac{d_{152}}{d_6 d_{21}^4 - d_7 d_{21}^2}; & d_{217} &= 6d_{172}^2 - 6d_{171} d_{173}; \\
 d_{218} &= 6d_{173} d_{172} - 6d_{174} d_{171}; & d_{219} &= d_{218} d_{171} - d_{217} d_{172}; & d_{220} &= \frac{d_{153}}{d_{171}}; & d_{221} &= \frac{-3d_{172} d_{153}}{d_{171}^2}; & d_{222} &= \\
 & \frac{d_{153} d_{217}}{d_{171}^3}; & d_{223} &= \frac{d_{153} d_{219}}{d_{171}^4}; & d_{224} &= 6d_{178}^2 - 6d_{179} d_{177}; & d_{225} &= 6d_{178} d_{179} - 6d_{180} d_{177}; & d_{226} &= \\
 & d_{225} d_{177} - d_{224} d_{178}; & d_{227} &= \frac{d_{154}}{d_{177}}; & d_{228} &= \frac{-3d_{178} d_{154}}{d_{177}^2}; & d_{229} &= \frac{d_{154} d_{224}}{d_{177}^3}; & d_{230} &= \frac{d_{154} d_{226}}{d_{177}^4}; & d_{231} &= d_{155} + \\
 & d_{177} + d_{162} d_{164}; & d_{232} &= d_{156} + d_{170}; & d_{233} &= d_{166} + d_{161}; & d_{234} &= d_{163} + d_{169}; & d_{235} &= d_{176} + d_{159} + \\
 & d_{204} + d_{223}; & d_{236} &= d_{160} + d_{182} + d_{208} + d_{230}; & d_{237} &= d_{157} + d_{187} + d_{196}; & d_{238} &= d_{158} + d_{192} + \\
 & d_{200}; & d_{239} &= d_{186} + d_{195}; & d_{240} &= d_{191} + d_{199}; & d_{241} &= d_{203} + d_{175} + d_{222}; & d_{242} &= d_{207} + d_{181} + d_{229}; \\
 & d_{243} &= d_{202} + d_{221}; & d_{244} &= d_{206} + d_{228};
 \end{aligned}$$

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