An Adaptive Observer Design for Nonlinear Systems Affected by Unknown Disturbance with Simultaneous Actuator and Sensor Faults. Application to a CSTR

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Abstract: Continuous Stirred Tank Reactor (CSTR) is an important system in the chemical and biological industries. It's characterized by a complex nonlinear behavior and is usually affected by faults and disturbances. Therefore, the states and faults estimation of a CSTR is always a challenging task for automated process researchers and engineers. This paper proposes an adaptive observer. This paper proposes an adaptive observer in order to estimate states and actuator and sensor faults simultaneously under unknown disturbance. Firstly, the approach of the Takagi-Sugeno multi-model is proposed to transform the complex nonlinear model into several simple linear sub-models. However, the states of the considered isotherm CSTR are not completely measurable, so the multi-model is represented with non-measurable premise variables. Then, in order to transform the considered system into a system with an unknown input, a mathematical transformation is introduced to describe the sensor faults as actuator faults. The proposed observer is designed, and the exponential stability conditions are studied with the Lyapunov theory and L2 optimization and formulated in terms of linear matrix inequalities. Finally, to improve the effectiveness of the proposed observer, a numerical simulation is carried out on a CSTR.

Keywords: CSTR; nonlinear system; multi-model approach; state estimation; faults estimation; adaptive observer; actuator fault; sensor fault.

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1. Introduction

Continuous Stirred Tank Reactor is used in biochemical and pharmaceutical processes in water treatment and several applications due to its appropriate mixing property. It is used for the perfect mixing of chemical components that are continuously added to the reactor in the chemical industry. The CSTR is characterized by complex nonlinear behavior and is exposed to faults and disturbances.

The complete knowledge of a system's state is necessary to construct a control law or a diagnosis mechanism. But, the state cannot be directly measurable due to economic or technical reasons. Therefore, state estimation becomes a principal step in process engineering. Observers have been used to generate a state estimation. An observer is based on the input and the measurable output to obtain an estimate of the state. The observer has first been proposed by Luenberger [1] for linear systems and later constructed for nonlinear systems [2].

Various approaches have been proposed to ensure the state estimation of a CSTR. In [3], a state estimation of a chemical reactor is proposed using the Extended Kalman Filter (EKF), Particle Filter (PF), and recently developed variational bayesian filter (VBF). In [4], the

Extended Kalman Filter (EKF) and the neural network (NN) based state filtering algorithm has been used to estimate the states variables of a CSTR. In [5], the states of a CSTR are estimated using Luenberger Observer, Sliding Mode Observers (SMO), and Kalman Observers (KO). However, these works are based directly on the nonlinear model of the CSTR, and from the mathematical point of view, an observed design based directly on the nonlinear model is very complex due to the high nonlinearity of the system behavior. The best solution to this problem is the Takagi-Sugeno (TS) multi-model approach [6]. It consists of representing the nonlinear model as an interpolation of simple linear models; each submodel presents the system's behavior on a limited part of the operating space.

In industry, the CSTR is usually exposed to faults, affecting the system's input as an actuator fault or the system's output as a sensor fault. In practice, actuator faults and sensor faults can occur simultaneously. Thus, the fault sensor may destroy the actuator due to the wrong measurement. For that purpose, the control and diagnosis framework requires knowledge of all the system states and the faults affecting them. Consequently, an observer is important to estimate these signals simultaneously.

Many works have been proposed observers for state estimation of a TS system under faults [7-24]. We can note some applications on chemical systems using the Fuzzy Luenberger observer [25,26], the Proportional-Integral observer (PIO) [27-29], the Unknown Input Observer (UIO) [30,31], the H ∞ observer [32,33], and sliding mode observer [34]. Most of these works supposed that the variable premises are measurable or coped with actuator faults without reference to sensor faults, making these methods less applicable.

This paper aims to design an adaptive observer for nonlinear systems described by Takagi-Sugeno with non-measurable decision variables and affected by the simultaneous actuator and sensor faults under unknown disturbances. The proposed technic is based on a proportional-integral observer with unknown input. Firstly, the multi-model structure is used to represent the nonlinear system by interpolating local linear models. Then, a mathematical transformation is used to rewrite the sensor fault as an actuator fault. The exponential stability conditions are synthesized using the Lyapunov function and L2 optimization and formulated in LMIs. To improve the efficacy of the proposed method, it is applied to a CSTR.

The main contributions of this work are: 1) using the multi-model approach to circumvent the nonlinearity of the system; 2) extending the adaptive observer to state and faults estimation for TS systems with unmeasurable premise variables; 3) the considered system is affected by the simultaneous actuator and sensor faults under unknown disturbances; 4) new stability conditions are obtained based on the Lyapunov theory and L2 optimization, and formulated in terms of LMIs which can be solved easily in Matlab LMI Toolbox; finally 5) application of the proposed method on a CSTR.

This paper is organized as follows: the Takagi-Sugeno approach is presented in Section 2, Section 3 concerns the problem statement and the adaptive observer synthesis, Section 4 focuses on the application to the CSTR system, finally, the conclusion is given in section 5.

2. Takagi-Sugeno multi-model approach

The Takagi-Sugeno multi-model method aims to divide the nonlinear system into an interpolation of linear sub-models. The multiple models are presented in the following form:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{l} \mu_i(\xi(t)) (A_i x(t) + B_i u(t)) \\ y(t) = \sum_{i=1}^{l} \mu_i(\xi(t)) C_i x(t) \end{cases}$$
(1)

where $x(t) \in \mathbb{R}^n$ is the state vector, $y(t) \in \mathbb{R}^p$ is the output, $u(t) \in \mathbb{R}^m$ is the control. l is the number of local models, $A_i \in \mathbb{R}^{n \times n}$, $B_i \in \mathbb{R}^{n \times m}$ and $C_i \in \mathbb{R}^{p \times n}$ are constant matrices of appropriate dimensions. $\mu_i(\xi(t))$ are the activation functions which define the activation degree of a local model and $\xi(t)$ is the decision variable. These functions satisfy the following convexity property:

$$\begin{cases} \sum_{i=1}^{l} \mu_i(\xi(t)) = 1, \ \forall t \\ 0 \le \mu_i(\xi(t)) \le 1, \ \forall i = 1 \dots l \end{cases}$$

$$\tag{2}$$

3. The design of the adaptive observer

This section discusses the synthesis of an adaptive observer for a Takagi-Sugeno multimodel system with non-measurable variables decision with simultaneous actuator and sensor faults.

3.1. Problem statement.

Let us consider a TS multi-model affected by actuator and sensor faults as follows: $\begin{cases} \dot{x}(t) = \sum_{i=1}^{l} \mu_i(x(t)) (A_i x(t) + B_i u(t) + E_i f_a(t) + wd(t)) \\ y(t) = C x(t) + S f_s(t) + \omega d(t) \end{cases}$ (3)

where $f_a(t)$ and $f_s(t)$ are respectively the actuator and sensor faults, and d(t) is an unknown disturbance. E_i and S are the faults distribution matrices that are supposed to be known.

A new state
$$q(t)$$
 is introduced in order to rewrite the sensor faults as actuator faults.
 $\dot{q}(t) = \sum_{i=1}^{l} \mu_i(x(t)) (-Dq(t) + Dy(t))$
(4)
The augmented system is presented as:

$$\begin{cases} \dot{X}(t) = \sum_{i=1}^{l} \mu_i (x(t)) (\bar{A}_i x(t) + \bar{B}_i u(t) + \bar{E}_i f(t) + \bar{w} d(t)) \\ Y(t) = \bar{C} x(t) \\ \text{where } \bar{w} = \begin{bmatrix} w & 0 \\ 0 & D \omega \end{bmatrix} \end{cases}$$
(5)

The states of the considered system are supposed not completely measurable. Therefore. (5) is rewritten as a multi-model with estimated variables decision.

$$\begin{cases} \dot{X}(t) = \sum_{i=1}^{l} \mu_i (\hat{\mathbf{x}}(t)) (\bar{A}_i x(t) + \bar{B}_i u(t) + \bar{E}_i f(t) + \bar{w} d(t)) + \Delta(t) \\ Y(t) = \bar{C} x(t) \end{cases}$$
(6)

where $\hat{x}(t)$ is the estimated x(t) by the proposed observer and $\Phi(t)$ given by:

$$\Delta(t) = \sum_{i=1}^{l} (\mu_i(x(t)) - \mu_i(\hat{x}(t))) (\bar{A}_i x(t) + \bar{B}_i u(t) + \bar{E}_i f(t) + \bar{w} d(t))$$
(7)

The system (6) can be rewritten as a faulty multi-model with unknown input.

$$\begin{cases} X(t) = \sum_{i=1}^{t} \mu_i(\hat{\mathbf{x}}(t)) (A_i \mathbf{x}(t) + B_i u(t) + E_i f(t)) + W \phi(t) \\ Y(t) = \bar{C} \mathbf{x}(t) \end{cases}$$
(8)
with

with

$$\begin{split} \bar{A}_i &= \begin{bmatrix} A_i & 0\\ DC & -D \end{bmatrix}; \ \bar{B}_i = \begin{bmatrix} B_i\\ 0 \end{bmatrix}; \ \bar{E}_i &= \begin{bmatrix} E_i & 0\\ 0 & DS \end{bmatrix}; \ \bar{C} &= \begin{bmatrix} C & 0\\ 0 & I \end{bmatrix}; \\ X(t) &= \begin{bmatrix} x(t)\\ q(t) \end{bmatrix}; \ f(t) &= \begin{bmatrix} f_a(t)\\ f_s(t) \end{bmatrix}; \\ \bar{W} &= [\bar{w} \quad I]; \ \varphi(t) &= \begin{bmatrix} d(t)\\ \Delta(t) \end{bmatrix}. \end{split}$$

3.2. Design and stability conditions of the adaptive observer.

The proposed observer is structured as follows:

$$\begin{cases} \dot{z}(t) = \sum_{i=1}^{l} \mu_i(\hat{x}(t)) (R_i z(t) + T\bar{B}_i u(t) + T\bar{E}_i \hat{f}(t) + K_{Pi} y(t)) \\ \dot{\hat{f}}(t) = \sum_{i=1}^{l} \mu_i(\hat{x}(t)) K_{Ii} (Y - \bar{C} \hat{X}) \\ \hat{X}(t) = z(t) + HY(t) \end{cases}$$
(9)

where $\hat{x}(t)$ is the estimated system state, z(t) is an intermediate variable and $\hat{f}(t)$ is the estimated faults. The variables R_i , T, K_{Pi} , K_{Ii} and H are the observer gains.

Stability conditions are obtained in the following theorem.

Theorem

The convergence of the state estimation error between the system (6) and the adaptive observer (9) is verified and the L2-gain of the transfer from $\Phi(t)$ to the estimation error is bounded if there exists $P = P^T > 0$ and matrix M_i and positive scalars $\overline{\gamma}$ the solution to the following optimization problem:

$$\min \bar{\gamma}$$

P, $M_i, \bar{\gamma}$

and the following conditions hold:

$$\begin{bmatrix} \bar{A}_{ai}^{T}P + P\bar{A}_{ai} - \bar{C}_{a}^{T}M_{i}^{T} - M_{i}\bar{C}_{a} + 2\alpha P + I & P\Gamma\\ \Gamma^{T}P & -\bar{\gamma}I \end{bmatrix} < 0 \ i = 1, \ \dots, \ l$$
(10)

with $\alpha > 0$ and:

$$\begin{bmatrix} T & H \end{bmatrix} = \begin{bmatrix} I_n \\ \bar{C} \end{bmatrix}^+ \tag{11}$$

$$\begin{bmatrix} F_i \\ K_{Ii} \end{bmatrix} = \overline{K}_i = X^{-1} M_i \tag{12}$$

$$R_i = \mathrm{T}\bar{A_i} - F_i\bar{C} \tag{13}$$

$$K_{Pi} = F_i + R_i M \tag{14}$$

and the L2-gain of the transfer is calculated by: $\gamma = \sqrt{\bar{\gamma}}$

Proof

e(t) and $e_f(t)$ represent the state and the fault estimation error respectively:

$$e(t) = X(t) - \hat{X}(t) \tag{15}$$

$$e_f(t) = f(t) - \hat{f}(t)$$
 (16)

The dynamics of the state estimation error is obtained as:

$$\dot{e}(t) = \dot{X}(t) - \hat{X}(t) = \sum_{i=1}^{l} \mu_i (\hat{x}(t)) ((P\bar{A}_i - R_i - F_i\bar{C})X(t) + (\Omega\bar{B}_i - T\bar{B}_i)u(t) + (\Omega\bar{E}_i - T\bar{E}_i)f(t) + \Omega\bar{W}\Phi(t) + R_ie(t)) + T\bar{E}_ie_f(t))$$
(17)

where

$$\Omega = I - H\bar{C} \tag{18}$$

$$F_i = K_{Pi} - R_i H \tag{19}$$

If the following conditions are verified

$$R_i = \Omega \bar{A}_i - F_i \bar{C}$$
(20)

$$\Omega = T$$
(21)

$$\Omega = T$$

The state estimation error can be rewritten as:

$$\dot{e}(t) = R_i e(t) + T \bar{E}_i e_f(t) + \Omega \bar{W} \Phi(t)$$
(22)

The dynamics of the fault estimation error is obtained by:

$$\dot{e}_f = -\hat{\bar{u}}(t) = -\sum_{i=1}^l \mu_i(\hat{x}(t)) K_{li}(Y - \bar{C}\hat{X}) = -\sum_{i=1}^l \mu_i(\hat{x}(t)) K_{li}\bar{C}e(t)$$
(23)
The augmented form of $\dot{e}(t)$ and $\dot{e}_f(t)$ is given by:

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$$\dot{e}_{a}(t) = \begin{bmatrix} R_{i} & T\bar{E}_{i} \\ -K_{Ii}\bar{C} & 0 \end{bmatrix} e_{a}(t) + \begin{bmatrix} \Omega \bar{W} \\ 0 \end{bmatrix} \Phi(t)$$
where $e_{a}(t) = \begin{bmatrix} e(t) \\ e_{u}(t) \end{bmatrix}$
(24)

The estimation error $e_a(t)$ is reduced to:

$$\dot{e}_{a}(t) = (\bar{A}_{ai} - \bar{K}_{i}\bar{C}_{a})e_{a}(t) + \Gamma\Phi(t)$$
with: $\bar{A}_{ai} = \begin{bmatrix} \Omega\bar{A}_{i} & T\bar{E}_{i} \\ 0 & 0 \end{bmatrix}; \bar{K}_{i} = \begin{bmatrix} F_{i} \\ K_{Ii} \end{bmatrix}; \bar{C}_{a} = [\bar{C} \quad 0]; \Gamma = \begin{bmatrix} \Omega\bar{W} \\ 0 \end{bmatrix}$
Tacking account of (18) and (21), we can write the following equation:
$$(25)$$

$$\mathbf{I} = \mathbf{T} + M\bar{C} = \begin{bmatrix} T & M \end{bmatrix} \begin{bmatrix} I_n \\ \bar{C} \end{bmatrix}$$
(26)

The matrix I_n is an identity matrix with appropriate dimension. Therefore, the gains T an H can be obtained as:

$$\begin{bmatrix} T & H \end{bmatrix} = \begin{bmatrix} I_n \\ \bar{C} \end{bmatrix}^+$$
where $\begin{bmatrix} I_n \end{bmatrix}^+$ the pseudo-inverse of $\begin{bmatrix} I_n \end{bmatrix}$.
(27)

 $V(t) = e_a^T(t)Pe_a(t)$ (28)

 $e_a(t)$ converge exponentially to zero if:

$$\exists X = X^T > 0, \ \alpha > 0: \dot{V}(t) + 2\alpha V(t) < 0$$
⁽²⁹⁾

The time derivative of V(t) is calculated as:

$$\dot{V}(t) = \sum_{i=1}^{l} \mu_i(\hat{\mathbf{x}}(t)) (e_a^T(t)) ((\bar{A}_{ai} - \bar{K}_i \bar{C}_a)^T P + X(\bar{A}_{ai} - \bar{K}_i \bar{C}_a) + 2\alpha P) e_a(t) + T(t) \Gamma^T P e_a(t) + e_a^T(t) P \Gamma \Phi(t))$$

$$(30)$$

The minimization of L2-gain of transfer from $\Phi(t)$ to the estimation error $e_a(t)$ is given by:

$$\frac{\|e_a(t)\|_2}{\|\Phi(t)\|_2} < \gamma, \|\Phi(t)\|_2 \neq 0$$
(31)

The estimation error converges exponentially toward zero and the L2-gain of transfer from $\Phi(t)$ to the estimation error $e_a(t)$ is bounded if the following condition is verified: $\dot{V}(t) + e_a^T(t)e_a(t) - \gamma^2 \Phi^T(t)\Phi(t) + 2\alpha V(t) < 0$ (32)

By replacing V(t) in (31) by its definition, the following inequality is obtained:

$$\begin{bmatrix} \bar{A}_{ai}^{T}P + P\bar{A}_{ai} - \bar{C}_{a}^{T}M_{i}^{T} - M_{i}\bar{C}_{a} + 2\alpha P + I & P\Gamma\\ \Gamma^{T}P & -\bar{\gamma}I \end{bmatrix} < 0 \ i = 1, \ \dots, \ l$$
(33)

4. Simulation results

Φ

4.1. CSTR system description.

Consider an isothermal CSTR with series-parallel reaction (Van der Vusse reaction) described by the following reactions [35]:



Figure 1. Isothermal Continuous Stirred-Tank Reactor.

The volume of the reactor V is assumed constant, and the feed stream contains only the component A. k_1 , k_2 and k_3 are the reaction rate constants, C_{Af} is the inlet concentration of the product A which will be converted to the desired product B.

The following set of differential equations describe the van der Vusse reaction scheme in an isothermal CSTR with actuator and sensor faults under unknown disturbances:

$$\begin{cases} \frac{dC_A}{dt} = \left(\frac{F}{V} + f_a(t)\right) \left(C_{Af} - C_A\right) - k_1 C_A - k_3 C_A^2 + w_1 d(t) \\ \frac{dC_B}{dt} = -\left(\frac{F}{V} + f_a(t)\right) C_B - k_1 C_A - k_2 C_B + w_2 d(t) \end{cases}$$
(34)

$$y(t) = C_B + Sf_S(t) + \omega d(t)$$
(35)

The equations for C_C and C_D are neglected because C_B is independent of them. The manipulated input in this system is the dilution rate $\frac{F}{V}$, and the controlled output variable is the product concentration C_B . The state representation of the system is as follows:

$$\begin{bmatrix} \dot{C}_A \\ \dot{C}_B \end{bmatrix} = \begin{bmatrix} -k_1 - k_3 C_A & 0 \\ k_1 & -k_2 \end{bmatrix} \begin{bmatrix} C_A \\ C_B \end{bmatrix} + \begin{bmatrix} C_{Af} - C_A \\ -C_B \end{bmatrix} \frac{F}{v} + \begin{bmatrix} C_{Af} - C_A \\ -C_B \end{bmatrix} f_a(t) + \begin{bmatrix} W_1 \\ W_2 \end{bmatrix} d(t)$$
(36)

Equation (36) can be written as: $\begin{cases}
\dot{x}(t) = A(x(t))x(t) + B(x(t))u(t) + B(x(t))f_a(t) + wd(t) \\
y(t) = Cx(t) + Sf_s(t) + \omega d(t)
\end{cases}$ (37) where $w(t) = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$

The parameters values are given in Table 1.

Table 1. Parameters values.			
Parameters	Values		
k_1	$\frac{5}{6}min^{-1}$		
k_2	$\frac{5}{3}min^{-1}$		
k_3	$\frac{1}{6}$ mol/l.min		
C_{Af}	10 mol/l		

4.2. Isothermal CSTR Takagi-Sugeno model design.

In the state representation (37), C_A and C_B are nonlinear terms, so we make them as our fuzzy variables. Generally, they are denoted by $z_i(t)$, and are known as premise variables.

Therefore, $z_1(t) = C_A$ and $z_2(t) = C_B$, $z_i(t)$ can be represented by membership functions F_i , f_i , the maximums M_i and the minimums m_i of $z_i(t)$ as follows:

$$\begin{cases} z_1(t) = F_1(z_1(t))M_1 + f_1(z_1(t))m_1 \\ z_2(t) = F_2(z_2(t))M_2 + f_2(z_2(t))m_2 \end{cases}$$
(38)

According to (38), the membership functions can be calculated as follows:

$$\begin{cases} F_1(z_1(t)) = \frac{z_1(t) - m_1}{M_1 - m_1}, & f_1(z_1(t)) = \frac{M_1 - z_1(t)}{M_1 - m_1} \\ F_2(z_2(t)) = \frac{z_2(t) - m_2}{M_2 - m_2}, & f_2(z_2(t)) = \frac{M_2 - z_2(t)}{M_2 - m_2} \end{cases}$$
(39)

and the activations function are:

$$\begin{cases} \mu_{1}(z(t)) = F_{1}(z_{1}(t))F_{2}(z_{2}(t)) \\ \mu_{2}(z(t)) = f_{1}(z_{1}(t))F_{2}(z_{2}(t)) \\ \mu_{3}(z(t)) = F_{1}(z_{1}(t))f_{2}(z_{2}(t)) \\ \mu_{4}(z(t)) = f_{1}(z_{1}(t))f_{2}(z_{2}(t)) \\ \text{The subsystems are determined as:} \\ \begin{cases} \dot{x}(t) = \sum_{i=1}^{4} \mu_{i}(z(t))[A_{i}x(t) + B_{i}u(t) + E_{i}f_{a}(t) + wd(t)] \\ y(t) = Cx(t) + Sf_{s}(t) + \omega d(t) \end{cases}$$
(40)

where $E_i = B_i$ and matrices A_i , B_i and C are given in Table 2.

Table 2. Sub-models of CSTR.			
Sub-model <i>i</i>	A_i	B_i	С
1	$\begin{bmatrix} -k_1 - k_3 M_1 \\ k_1 \end{bmatrix}$	$\begin{bmatrix} C_{Af} - M_1 \\ -M_2 \end{bmatrix}$	
2	$\begin{bmatrix} -k_1 - k_3 m_1 \\ k_1 \end{bmatrix}$	$\begin{bmatrix} C_{Af} - m_1 \\ -M_2 \end{bmatrix}$	[0 1]
3	$\begin{bmatrix} -k_1 - k_3 M_1 \\ k_1 \end{bmatrix}$	$\begin{bmatrix} C_{Af} - M_1 \\ -m_2 \end{bmatrix}$	
4	$\begin{bmatrix} -k_1 - k_3 m_1 \\ k_1 \end{bmatrix}$	$\begin{bmatrix} \mathcal{C}_{Af} - m_1 \\ -m_2 \end{bmatrix}$	

 Table 2.
 Sub-models of CSTR.

4.3. Simulations results and discussions.

The acting faults appear and disappear during an interval time as follows: $f_a = \begin{cases} 0.4, & 20 \le t \le 50 \\ 0 & \text{elsewhere} \end{cases}; f_s = \begin{cases} 0.3 \sin(0.06\pi t), & 35 \le t \le 70 \\ 0 & \text{elsewhere} \end{cases}$

The unknown disturbances are presented by zero-mean noise with standard deviations equal to 0.05. The distribution matrices S, W and ω are given by:

 $S = 0.75; W = 1.2 \text{ and } \omega = \begin{bmatrix} 0.8\\ 0.9 \end{bmatrix}$



Figure 2. Time-evolution of the weighting functions.

The LMI elaborated in the proposed theorem is solved using the Matlab Yalmip toolbox, and the gains of the adaptive observer are calculated. States and faults estimations are presented in Figure 5 and Figure 6.



The weighting functions for each sub-model are presented in Figure 2, always close to the dynamic behavior of the nonlinear system according to the considered operating regimes. Figure 3 and Figure 4 present the control input and the unknown disturbance signals, respectively. Figure 5 shows that from moment zero, the estimated states converge rapidly to their real signals. Figure 6 presents a good estimation of the actuator and sensor faults, respectively. According to the simulation results, we can clearly see that the proposed observer ensures the estimation of states and the simultaneous actuator and sensor faults despite unknown disturbances.

5. Conclusions

In this work, an adaptive observer is proposed to estimate the states and simultaneous actuator and sensor faults of a CSTR with unknown disturbances. Firstly, the Takagi-Sugeno method is proposed to represent the complex nonlinear system as a multi-model with unmeasurable variables decision. Secondly, a mathematical transformation is used to rewrite all faults as actuator faults. The exponential stability conditions are studied with Lyapunov theory and L2 optimization and given in linear matrix inequalities form. Finally, a simulation example is carried out on a CSTR. From the results, states, actuator, and sensor faults are successfully estimated in the presence of the unknown disturbances by minimizing the effects of these last.

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Conflicts of Interest

The authors declare no conflict of interest.

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