

MHD Free Convection from a Semi-infinite Vertical Porous Plate with Diffusion-Thermo Effect

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Abstract: An analytical solution for two-dimensional unsteady MHD free convective mass transfer flows of viscous incompressible optically thin fluid past a semi-infinite vertical porous plate in the presence of thermal radiation and chemical reaction is presented in this paper. A uniform magnetic field is applied normally to the plate with a first-order chemical reaction. The non-dimensional governing equations are solved analytically by using the regular perturbation technique. The effects of various physical parameters like radiation parameter Q , Dufour effect Du , chemical reaction parameter K , thermal Grashof number Gr , Hartmann number M , porosity parameter k , etc., are studied and demonstrated graphically. One of the significant findings of this analysis includes that an intensification of the chemical reaction effect causes a downfall in the fluid concentration. In contrast, another important outcome of the present study is that the rate of heat transfer and shear stress at the wall increases under the diffusion thermo effect or Dufour effect. Still, it tends to fall for high radiation. Further, the rate of mass transfer rises under the chemical reaction effect.

Keywords: MHD; Dufour effect; thermal radiation; chemical reaction; porosity.

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Nomenclature:

C Species concentration of the fluid; $K.mol.m^{-3}$	\bar{u} Component of velocity along x -axis; $m.s^{-1}$
\bar{C}_∞ Species concentration of the fluid far away from the plate; $K.mol.m^{-3}$	\bar{v} Component of velocity along y -axis; $m.s^{-1}$
\bar{C}_w Species concentration of the fluid near the plate; $K.mol$	v Velocity of the fluid; $m.s^{-1}$
C_p Specific heat at constant pressure; $J.kg^{-1}.K^{-1}$	v_0 Scale of the suction velocity; $m.s^{-1}$
D_M Chemical molecular diffusivity; $m^2.s^{-1}$	Greek symbols
ρ Density of the fluid; $kg.m^{-3}$	μ Coefficient of viscosity; $kg.m^{-1}.s^{-1}$
g Acceleration due to gravity; $m.s^{-2}$	κ Thermal conductivity; $W.m^{-1}.K^{-1}$
q_r Radiative heat flux; $W.m^{-2}$	σ Electrical conductivity; $\frac{1}{(Ohm \times m)}$
T Temperature of the fluid; K	ν Kinematic viscosity; $m^2.s^{-1}$

\bar{T}_∞ Temperature far away from the plate; K

β Coefficient of volume expansion; $J.kg^{-1}.K^{-1}$

\bar{T}_w Temperature of the plate; K

\bar{t} Time

1. Introduction

For several decades, magnetohydrodynamics (MHD) has attracted numerous scientists and engineers because of its fascination and importance in various technological devices and in understanding diverse cosmic phenomena. MHD concepts are used in designing heat exchangers, pumps, flow meters, and power generation systems, and confinement schemes for controlled fusion. MHD convection problems are extremely important in the study of aeronautics, particularly in missile aerodynamics, because the temperatures that occur at such high speeds are sufficient to dissociate or even ionize the air significantly. MHD flow has applications in metrology, solar physics, and the motion of the 'earth's core. The concepts of MHD are widely used in medicine and biology. Problems involving convection with hydromagnetic fluxes are also important in geophysics, astrophysics, plasma physics, missile technology, and other fields. The current form of MHD is based on the pioneering contributions of many authors Alfven [1], Cowling [2], Shercliff [3], Ferraro and Plumpton [4], and Crammer and Pai [5]. Several authors have contributed to MHD in recent times; some of them are Raghunath *et al.* [6], Anwar *et al.* [7], Gaud *et al.* [8], Sheri *et al.* [9], Omamoke *et al.* [10], Seth *et al.* [11], etc.

Natural convection is a heat transfer phenomenon in which the driver of the fluid motion is self-induced forces. These forces may be due to temperature or concentration gradients. It has many applications in chemical engineering, petroleum technology, agricultural engineering, etc. Recently, Ahmed and Choudhury [12], Sinha and Ahmed [13], Allan and Dardery [14], Chamuah *et al.* [15], Ahmed and Dutta [16], Mahato *et al.* [17], and many authors have studied MHD free convective heat and mass transfer flow through a porous medium.

The phenomenon by which heat transfer is caused by concentration gradient is called diffusion-thermo or Dufour effect. The concentration gradient results in a temperature change. The effect was first observed by the Swiss physicist L. Dufour in 1873. It has enormous applications in many fields, especially in CVD problems and chemical reactors. Recently, Dagana and Amos [18] investigated MHD free convection heat and mass transfer flow in a porous medium with Dufour and chemical reaction effects. Reddy [19] studied Dufour's effects on heat and mass transfer of non-Newtonian power-law fluid over a porous plate. Ahmed *et al.* [20] examined three-dimensional hydromagnetic convective flow past a porous vertical plate with sinusoidal suction in slip flow regime, taking the Soret and Dufour effect into account. Jha and Sarki [21] worked on Chemical reaction and Dufour effects on nonlinear free convection heat and mass transfer flow near a vertical moving porous plate. Dufour's effect on MHD free convection heat and mass transfer flow over an inclined plate embedded in a porous medium was investigated by Islam *et al.* [22].

In recent times, many researchers have given considerable interest to the study of MHD in the presence of radiation. Mishra *et al.* [23] studied the effect of radiation and non-uniform heat source on the unstable, MHD viscous fluid through a heated plate with time-dependent suction and viscous dissipation. Chiranjeevi *et al.* [24] investigated the problem of the MHD boundary layer flow analysis in the presence of thermal absorption with heat generation and chemical reaction over a plate. To simulate the 2D unstable magneto-hydrodynamic flow of an

electrically conducting fluid over a permeable plate under the effect of thermal radiation and chemical reaction, a mathematical model has been developed by Suneetha *et al.* [25]. Gaud [26] examined the impacts of thermal radiation on the MHD stagnation point stream over a stretching sheet with slip boundary conditions.

In chemical engineering, chemical reactions have always played an important role in heat and mass transfer. A theoretical analysis has been done by Rajput and Kumar [27] to study the unsteady MHD flow through porous medium past an exponentially accelerated vertical plate with variable wall temperature and mass diffusion in the presence of Hall current, radiation, and chemical reaction. Thermal radiation on unsteady electrical MHD flow of nanofluid overstretching sheet with chemical reaction was investigated by Daniel *et al.* [28]. Reddy *et al.* [29] investigated the effects of chemical reaction on unsteady MHD flow past an impulsively started oscillating infinite vertical plate with variable temperature and constant mass diffusion in the presence of Hall current. Suneetha *et al.* [30] studied the combined effects of thermal radiation and chemical reaction on steady MHD mixed convective heat and mass transfer flow past a vertical surface under the influence of Joule and viscous dissipation. Rajakumar *et al.* [31] studied radiation, dissipation, and Dufour effects on MHD free convection Casson fluid flow through a vertical oscillatory porous plate with the ion-slip current. Obulesu *et al.* [32] discussed current hall effects on MHD convective flow past a porous plate with thermal radiation, chemical reaction with radiation absorption. Recently, an analytical solution based on the asymptotic series expansion method is presented by Ahmed and Bordoloi [33] for the problem of a fully developed chemically reactive laminar three-dimensional flow through a porous vertical infinite plate in a porous medium sinusoidal permeability as suction in the existence of appreciable radiation. Seth *et al.* [34] investigated unsteady MHD flow of a Casson fluid near a vertical oscillating plate through a non-Darcy porous medium. Kumar *et al.* [35] explained the effect of magnetite nanofluid, taking into account water as the base fluid, over a rotating disk in the presence of the external magnetic field.

Because of the above studies, this paper investigates the effect of thermal radiation and chemical reaction on MHD free convective mass transfer flow of viscous incompressible optically thin fluid with Dufour effect using Cogley *et al.* [36] model. The novelty of the present investigation is that the present problem deals with a flow through a porous medium of Brickman type in the presence of a transverse magnetic field. Rosseland type of radiation, diffusion-thermo effect, and homogenous first-order chemical reaction are also considered in our study's preview. The asymptotic series expansion method is utilized for getting the solution of the governing equations. It is worthwhile to mention that asymptotic series expansion is unconditionally convergent. The Present investigation may impact various engineering processes such as paper production, glass blowing, extrusion of plastic sheets, etc.

2. Materials and Methods

2.1. Basic equations.

Equation of continuity:

$$\vec{\nabla} \cdot \vec{q} = 0 \tag{1}$$

Momentum equation:

$$\rho \left[\frac{\partial \vec{q}}{\partial t} + (\vec{q} \cdot \nabla) \vec{q} \right] = \vec{F} - \nabla \bar{p} + \mu \nabla^2 \vec{q} + \vec{J} \times \vec{B} - \mu \frac{\vec{q}}{k} \quad (2)$$

Energy equation:

$$\rho C_p \left[\frac{\partial \bar{T}}{\partial t} + (\vec{q} \cdot \nabla) \bar{T} \right] = \kappa \nabla^2 \bar{T} - \nabla \cdot \vec{q}_r + \frac{\rho D_M K_T}{C_s} \nabla^2 \bar{C} \quad (3)$$

Species continuity equation:

$$\frac{\partial \bar{C}}{\partial t} + (\vec{q} \cdot \nabla) \bar{C} = D_M \nabla^2 \bar{C} + K^* (\bar{C}_\infty - \bar{C}) \quad (4)$$

2.2. Mathematical formulation.

A cartesian coordinate system $(\bar{x}, \bar{y}, \bar{z})$ with the X-axis along with the infinite vertical plate, the Y-axis normal to the plate, and the Z-axis alongside the width of the plate has been considered. Initially, the plate and the fluid were at the same temperature \bar{T}_∞ with concentration levels \bar{C}_∞ at all points. At the time, $\bar{t} > 0$, the plate temperature was suddenly raised \bar{T}_w , and the concentration level at the plate rose \bar{C}_w . A uniform magnetic field is applied normally to the plate.

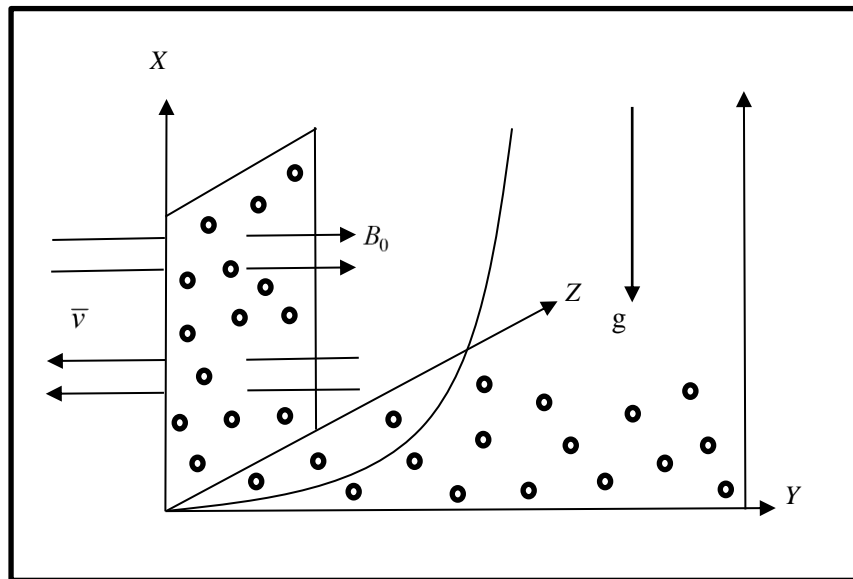


Figure 1. Physical configuration.

To make the mathematical model of the theoretical problem idealized, the following constraints are imposed:

- (a) The fluid is incompressible, viscous electrically conducting.
- (b) All the fluid properties are constant, excluding density in the buoyancy force term.
- (c) External electric field is negligible.
- (d) Ohmic dissipation, as well as viscous dissipation, is not considered.
- (e) Induced magnetic field in comparison to the applied magnetic field is negligible.
- (f) No external electric field is applied, for which $\vec{E} = 0$

Gauss's law of magnetism

$$\text{Div } \vec{B} = 0 \quad (5)$$

Electric current density

$$\vec{J} = \sigma[\vec{E} + \vec{q} \times \vec{B}] \tag{6}$$

$\vec{B} = B_0 \hat{j}$, is the uniform magnetic field with strength B_0 applied normal to the plate, directed into the fluid region. Due to semi-infinite plate surface assumptions, all the flow variables except pressure are functions of \bar{y} and \bar{t} only.

Thus, the governing equations take the forms as given below:

A. Continuity equation

$$\frac{\partial \bar{v}}{\partial \bar{y}} = 0 \tag{7}$$

It is seen from equation (7) that \bar{v} it is a constant. Assuming the suction velocity to be oscillatory, we have

$$\bar{v} = -v_0(1 + \varepsilon A e^{i\omega t}) \tag{8}$$

Where ε, A are small such that $\varepsilon A \ll 1$. The negative sign indicates that the suction velocity is directed towards the plate.

X- component of momentum equation

$$\rho \left[\frac{\partial \bar{u}}{\partial \bar{t}} + (\bar{v} \frac{\partial}{\partial \bar{y}}) \bar{u} \right] = -\rho g - \frac{\partial \bar{p}}{\partial \bar{x}} + \mu \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} - \sigma B_0^2 \bar{u} - \frac{\mu \bar{u}}{k} \tag{9}$$

In the free stream, equation (8) takes the form

$$0 = -\rho_\infty g - \frac{\partial \bar{p}}{\partial \bar{x}} \tag{10}$$

Subtracting (10) from (9), we obtain

$$\rho \left[\frac{\partial \bar{u}}{\partial \bar{t}} + \bar{v} \frac{\partial \bar{u}}{\partial \bar{y}} \right] = (\rho_\infty - \rho) g + \mu \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} - \sigma B_0^2 \bar{u} - \frac{\mu \bar{u}}{k} \tag{11}$$

Volume expression gives

$$\rho_\infty - \rho = \rho \left[\beta (\bar{T} - \bar{T}_\infty) + \beta^* (\bar{C} - \bar{C}_\infty) \right] \tag{12}$$

Using (12) in (11), we get

B. \bar{x} - component of momentum equation [12],

$$\frac{\partial \bar{u}}{\partial \bar{t}} + \bar{v} \frac{\partial \bar{u}}{\partial \bar{y}} = g \beta (\bar{T} - \bar{T}_\infty) + g \beta^* (\bar{C} - \bar{C}_\infty) + \nu \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} - \frac{\sigma B_0^2 \bar{u}}{\rho} - \frac{\nu \bar{u}}{k} \tag{13}$$

Now the conservation of energy equation [21],

$$\frac{\partial \bar{T}}{\partial \bar{t}} + \bar{v} \frac{\partial \bar{T}}{\partial \bar{y}} = \frac{\kappa}{\rho C_p} \frac{\partial^2 \bar{T}}{\partial \bar{y}^2} - \frac{1}{\rho C_p} \frac{\partial q_r}{\partial \bar{y}} + \frac{D_M K_T}{C_p C_S} \frac{\partial^2 \bar{C}}{\partial \bar{y}^2} \tag{14}$$

According to Cogley's model, the radiative heat flux in optically thin non-Gray gas near equilibrium is specified by

$$\frac{\partial q_r}{\partial \bar{y}} = 4I(\bar{T} - \bar{T}_\infty), \text{ where } I = \int_0^\infty (k_\lambda)_w \left(\frac{\partial e_{\lambda b}}{\partial \bar{T}} \right)_w d\lambda \tag{15}$$

Unification of (15) and (14) leads

C. Energy equation [12, 21],

$$\frac{\partial \bar{T}}{\partial \bar{t}} + \bar{v} \frac{\partial \bar{T}}{\partial \bar{y}} = \frac{\kappa}{\rho C_p} \frac{\partial^2 \bar{T}}{\partial \bar{y}^2} - \frac{1}{\rho C_p} 4I(\bar{T} - \bar{T}_\infty) + \frac{D_M K_T}{C_p C_S} \frac{\partial^2 \bar{C}}{\partial \bar{y}^2} \tag{16}$$

D. Mass transfer equation [10],

$$\frac{\partial \bar{C}}{\partial t} + \bar{v} \frac{\partial \bar{C}}{\partial \bar{y}} = D_M \frac{\partial^2 \bar{C}}{\partial \bar{y}^2} + K^* (\bar{C}_\infty - \bar{C}) \tag{17}$$

The corresponding boundary conditions are

$$\left. \begin{aligned} t > 0, \bar{u} = 0, \bar{T} = \bar{T}_\infty + (\bar{T}_w - \bar{T}_\infty)(1 + \varepsilon e^{i\omega t}), \bar{C} = \bar{C}_w \text{ at } \bar{y} = 0 \\ \bar{u} \rightarrow 0, \bar{T} \rightarrow \bar{T}_\infty, \bar{C} \rightarrow \bar{C}_\infty \text{ as } \bar{y} \rightarrow \infty \end{aligned} \right\} \tag{18}$$

To make the mathematical model normalized, we introduce the following non-dimensional quantities:

$$\left. \begin{aligned} y = \frac{v_0 \bar{y}}{\nu}, u = \frac{\bar{u}}{v_0}, t = \frac{\bar{t} v_0^2}{4\nu}, \theta = \frac{\bar{T} - \bar{T}_\infty}{\bar{T}_w - \bar{T}_\infty}, \phi = \frac{\bar{C} - \bar{C}_\infty}{\bar{C}_w - \bar{C}_\infty}, Sc = \frac{\nu}{D_M}, v = \frac{\bar{v}}{v_0}, \\ Gr = \frac{g\beta\nu(\bar{T}_w - \bar{T}_\infty)}{v_0^3}, Gc = \frac{g\beta^* \nu(\bar{C}_w - \bar{C}_\infty)}{v_0^3}, \omega = \frac{4\nu\bar{\omega}}{v_0^2}, M = \frac{\sigma B_0^2 \nu}{\rho v_0^2}, \\ k = \frac{\bar{k} v_0^2}{\nu^2}, K = \frac{K^* \nu}{v_0^2}, Du = \frac{D_M K_T (\bar{C}_w - \bar{C}_\infty)}{\nu C_p C_S (\bar{T}_w - \bar{T}_\infty)}, Pr = \frac{\mu C_p}{\kappa}, Q = \frac{4I\nu}{\rho C_p v_0^2} \end{aligned} \right\} \tag{19}$$

Under the transformations (19), the governing equations (13), (16), (17) reduces to the following forms

$$\frac{1}{4} \left(\frac{\partial u}{\partial t} \right) - (1 + \varepsilon A e^{i\omega t}) \frac{\partial u}{\partial y} = Gr \theta + Gc \phi + \frac{\partial^2 u}{\partial y^2} - \left(M + \frac{1}{k} \right) u \tag{20}$$

$$\frac{1}{4} \left(\frac{\partial \phi}{\partial t} \right) - (1 + \varepsilon A e^{i\omega t}) \frac{\partial \phi}{\partial y} = \frac{1}{Sc} \frac{\partial^2 \phi}{\partial y^2} - K \phi \tag{21}$$

$$\frac{1}{4} \left(\frac{\partial \theta}{\partial t} \right) - (1 + \varepsilon A e^{i\omega t}) \frac{\partial \theta}{\partial y} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial y^2} - Q \theta + Du \frac{\partial^2 \phi}{\partial y^2} \tag{22}$$

With conditions:

$$\left. \begin{aligned} t > 0, u = 0, \theta = \theta_w = 1 + \varepsilon A e^{i\omega t}, \phi = 1 \text{ at } y = 0 \\ u \rightarrow 0, \theta \rightarrow 0, \phi \rightarrow 0 \text{ as } y \rightarrow \infty \end{aligned} \right\} \tag{23}$$

2.3. Method of solutions.

To solve the equations (20), (21), and (22) under the conditions (23), we consider the following asymptotic form

$$\left. \begin{aligned} u(y, t) = u_0(y) + \varepsilon e^{i\omega t} u_1(y) + O(\varepsilon^2) \\ \theta(y, t) = T_0(y) + \varepsilon e^{i\omega t} T_1(y) + O(\varepsilon^2) \\ \phi(y, t) = C_0(y) + \varepsilon e^{i\omega t} C_1(y) + O(\varepsilon^2) \end{aligned} \right\} \tag{24}$$

Substituting (24) in the equations (20), (21), (22) under the condition (23) and equating the harmonic and non-harmonic terms, and also neglecting $O(\varepsilon^2)$, following differential equations are obtained.

$$u_0'' + u_0' - \left(M + \frac{1}{k} \right) u_0 = -Gr \theta_0 - Gc \phi_0 \tag{25}$$

$$u_1'' + u_1' - \left(M + \frac{1}{k} + \frac{i\omega}{4} \right) u_1 = -A u_0' - Gr \theta_1 - Gc \phi_1 \tag{26}$$

$$\theta_0'' + \text{Pr} \theta_0' - \text{Pr} Q \theta_0 = -\text{Pr} Du \phi_0'' \tag{27}$$

$$\theta_1'' + \text{Pr} \theta_1' - \left(Q + \frac{i\omega}{4} \right) \text{Pr} \theta_1 = -A \text{Pr} \theta_0' - \text{Pr} Du \phi_1'' \tag{28}$$

$$\phi_0'' + Sc \phi_0' - Sc K \phi_0 = 0 \tag{29}$$

$$\phi_1'' + Sc \phi_1' - \left(K + \frac{i\omega}{4} \right) Sc \phi_1 = -Sc A \phi_0' \tag{30}$$

Subject to the conditions:

$$\left. \begin{aligned} t > 0, u_0 = u_1 = 0, \theta_0 = \theta_1 = 1, \phi_0 = 1, \phi_1 = 0, \text{ at } y=0 \\ u_0 = u_1 \rightarrow 0, \theta_0 = \theta_1 \rightarrow 0, \phi_0 = \phi_1 \rightarrow 0 \text{ as } y \rightarrow \infty \end{aligned} \right\} \tag{31}$$

Solving equations (25) to (30) by using perturbation technique and using the boundary conditions (31), the solution for velocity, temperature, and concentration, respectively, are expressed as

$$u = (A_6 - A_7)e^{-m_5y} - A_6e^{-m_3y} + A_7e^{-m_1y} + \varepsilon e^{i\omega t} [(-A_8 - A_9 - A_{10} + A_{11} - A_{12})e^{-m_6y} + A_8e^{-m_5y} + A_9e^{-m_3y} + A_{10}e^{-m_1y} - A_{11}e^{-m_4y} + A_{12}e^{-m_2y}] \tag{32}$$

$$\theta = (1 + A_2)e^{-m_3y} - A_2e^{-m_1y} + \varepsilon e^{i\omega t} [(1 + A_3 + A_4 - A_5)e^{-m_4y} - A_3e^{-m_3y} - A_4e^{-m_1y} + A_5e^{-m_2y}] \tag{33}$$

$$\phi = e^{-m_1y} + A_1 \varepsilon e^{i\omega t} (e^{-m_1y} - e^{-m_2y}) \tag{34}$$

The non-dimensional skin friction τ at the plate $y=0$ is given by

$$\begin{aligned} \tau &= \left. \frac{\partial u}{\partial y} \right]_{y=0} \\ &= -m_5(A_6 - A_7) + m_3A_6 - m_1A_7 + \varepsilon e^{i\omega t} [-(-A_8 - A_9 - A_{10} + A_{11} - A_{12})m_6 \\ &\quad - m_5A_8 - m_3A_9 - m_1A_{10} + m_4A_{11} - m_2A_{12}] \end{aligned} \tag{35}$$

The rate of heat transfer at the plate in terms of the Nusselt number based on Fourier's law of heat conduction is expressed as

$$\begin{aligned} Nu &= \left. \frac{\partial \theta}{\partial y} \right]_{y=0} \\ &= -(1 + A_2)m_3 + A_2m_1 + \varepsilon e^{i\omega t} [-(1 + A_3 + A_4 - A_5)m_4 + A_3m_3 + A_4m_1 - A_5m_2] \end{aligned} \tag{36}$$

The rate of concentration in terms of Sherwood number is given by the relation

$$\begin{aligned} Sh &= \left. \frac{\partial \phi}{\partial y} \right]_{y=0} \\ &= -m_1 + A_1 \varepsilon e^{i\omega t} (m_2 - m_1) \end{aligned} \tag{37}$$

3. Results and Discussion

To get the physical insight into the problem, the effects of various parameters like radiation parameter Q , Dufour effect Du , Schmidt number Sc , thermal Grashof number Gr , Hartmann number M , porosity parameter k , etc. are studied graphically on velocity, temperature, concentration, skin friction, Nusselt number and Sherwood number which are portrayed from Figures 2-14. The default values for emerging flow parameters are chosen as

$Pr=0.71$, $Sc=0.7$, $Gr=5$, $Gc=5$, $k=15$, $K=5$, $Du=10$, $Q=5$, $M=10$, $\varepsilon = 0.01$, $A=0.01$, $\omega = 2$, $t = 0.2$ until otherwise specified particularly.

The effects of Du , Gr , and M on velocity profiles are displayed in Figures 2, 3, and 4, respectively. Figures 2 and 3 shows that fluid velocity increases with the increasing values of Du and Gr . Thus, the velocity of the fluid gets accelerated under diffusion thermo effect and thermal buoyancy force. From Figure 4, we see that the transverse magnetic field M tends to retard the fluid motion. This is because of the Lorentz force, which decelerates the fluid motion.

The variations in temperature versus normal coordinate y are demonstrated in figures 5 to 7. Figure 5 predicts a clear rise in the fluid temperature under diffusion thermal effect Du . Also, it is observed from Figure 6 that a rise in the positive values of radiation parameter Q results in a significant downfall of the magnitude of the temperature. Figure 7 shows that fluid temperature increases under chemical reaction parameters in a thin layer near the plate, and thereafter this behavior takes a reverse turn.

The effect of chemical reaction parameter K and Schmidt number Sc on the concentration is shown in Figures 8 and 9, respectively. It is seen that the concentration of the species decreases substantially due to the rise in chemical reaction effect and Schmidt number.

It is observed in Figure 10 that viscous drag gets accelerated for increasing the Dufour number Du . Thus, the diffusion-thermo effect has a considerable contribution in enhancing the frictional resistance on the plate. Whereas, reverse nature is seen for radiation Q in Figure 11. Figure 11 shows that viscous drag at the plate decreases as the radiation increases.

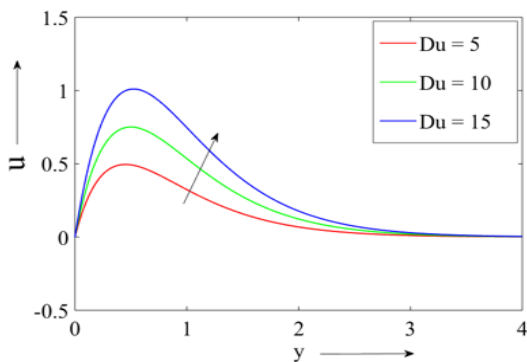


Figure 2. Velocity u vs y for variations in Du when $Pr=0.71$, $Sc=0.7$, $k=15$, $K=5$, $Q=5$, $Gr=5$, $Gc=5$, $M=10$, $\varepsilon = 0.01$, $A=0.01$, $\omega = 2$, $t=0.2$.

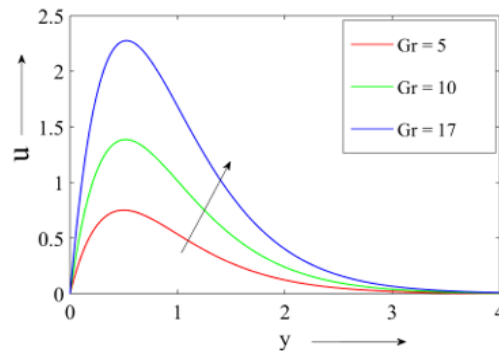


Figure 3. Velocity u vs y for variations in Gr when $Pr=0.71$, $Sc=0.7$, $k=15$, $K=5$, $Q=5$, $Du=10$, $Gc=5$, $M=10$, $\varepsilon = 0.01$, $A=0.01$, $\omega = 2$, $t=0.2$.

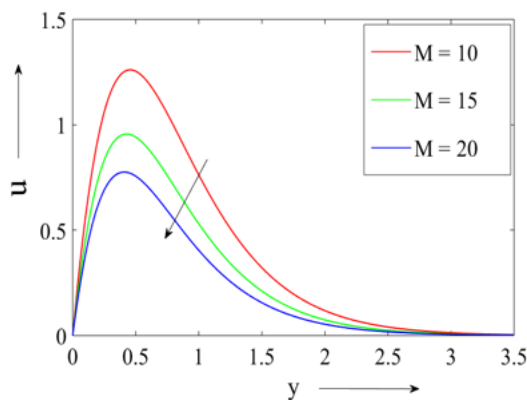


Figure 4. Velocity u vs y for variations in M when $Pr=0.71$, $Sc=0.7$, $k=15$, $K=20$, $Q=5$, $Du=10$, $Gr=5$, $Gc=5$, $\varepsilon = 0.01$, $A=0.01$, $\omega = 2$, $t=0.2$.

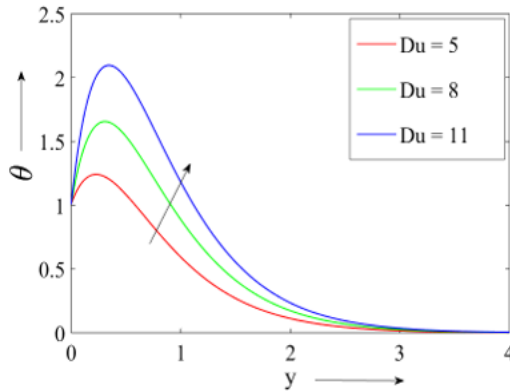


Figure 5. Temperature θ vs y for variations in Du when $Pr=0.71$, $Sc=0.7$, $K=5$, $Q=5$, $\varepsilon = 0.01$, $A=0.01$, $\omega = 2$, $t=0.2$.

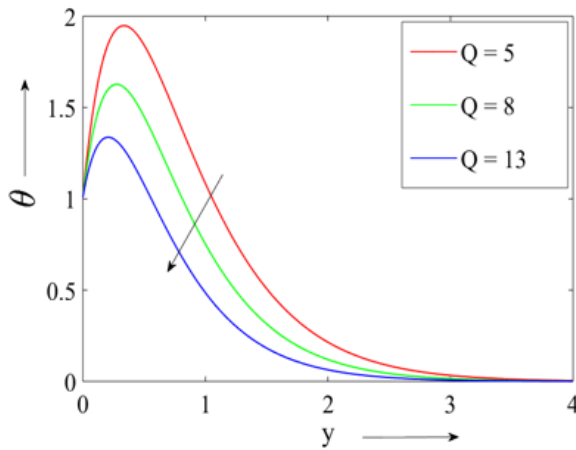


Figure 6. Temperature θ vs y for variations in Q when $Pr=0.71, Sc=0.7, K=5, Du=10, \varepsilon = 0.01, A=0.01, \omega = 2, t=0.2$.

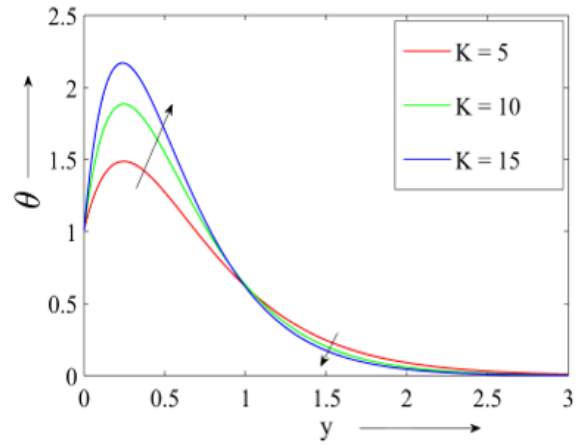


Figure 7. Temperature θ vs y for variations in K when $Pr=0.71, Sc=0.7, Q=5, Du=10, \varepsilon = 0.01, A=0.01, \omega = 2, t=0.2$.

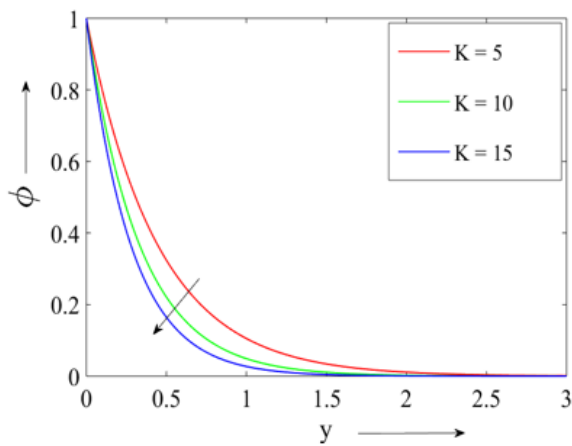


Figure 8. Concentration ϕ vs y for variations in K when $Sc=0.7, \varepsilon = 0.01, A=0.01, \omega = 2, t=0.2$.

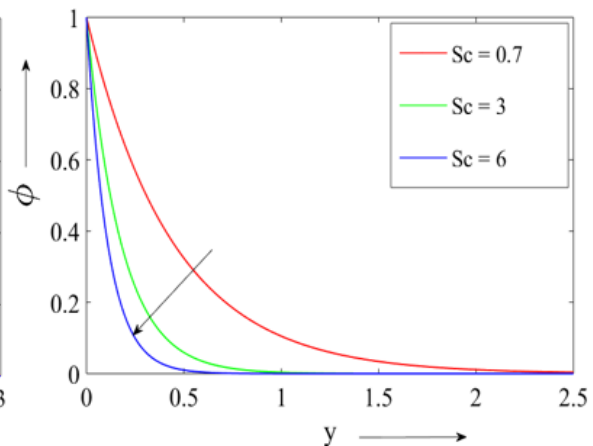


Figure 9. Concentration ϕ vs y for variations in Sc when $K=5, \varepsilon = 0.01, A=0.01, \omega = 2, t=0.2$.

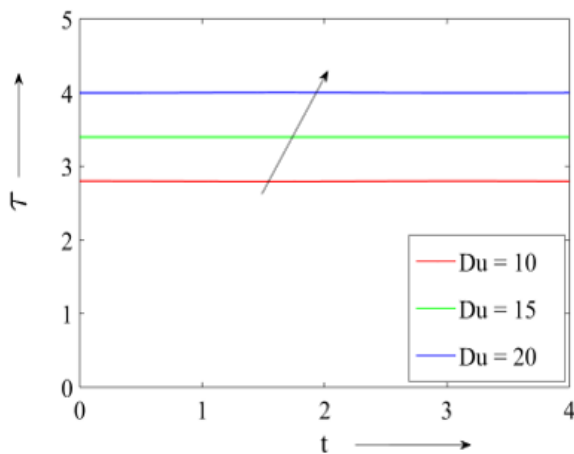


Figure 10. Skin friction τ vs t for variations in Du when $Pr=0.71, Sc=0.7, k=15, K=5, Q=5, Gr=5, Gc=5, M=10, \varepsilon = 0.01, A=0.01, \omega = 2$.

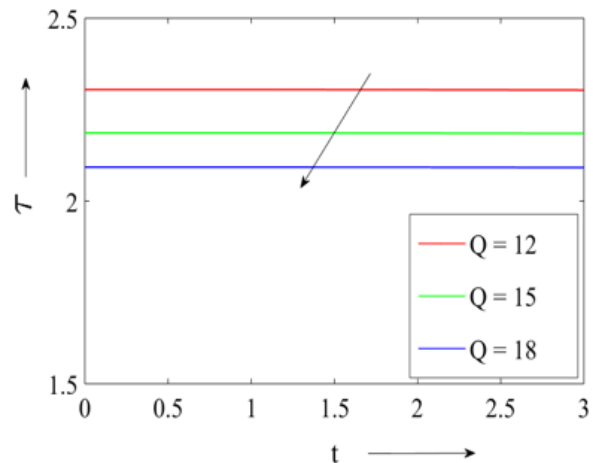


Figure 11. Skin friction τ vs t for variations in Q when $Pr=0.71, Sc=0.7, k=15, K=5, Du=10, Gr=5, Gc=5, M=10, \varepsilon = 0.01, A=0.01, \omega = 2$.

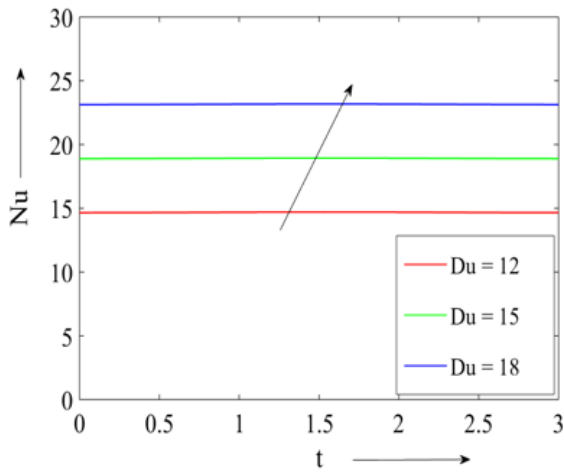


Figure 12. Nusselt number Nu vs t for variations in Du when $Pr=0.71$, $Sc=0.7$, $K=5$, $Q=5$, $\varepsilon = 0.01$, $A=0.01$, $\omega = 2$.

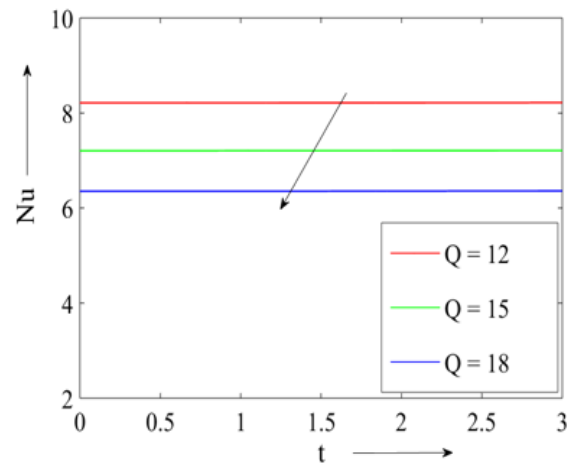


Figure 13. Nusselt number Nu vs t for variations in Q when $Pr=0.71$, $Sc=0.7$, $K=5$, $Du=10$, $\varepsilon = 0.01$, $A=0.01$, $\omega = 2$.

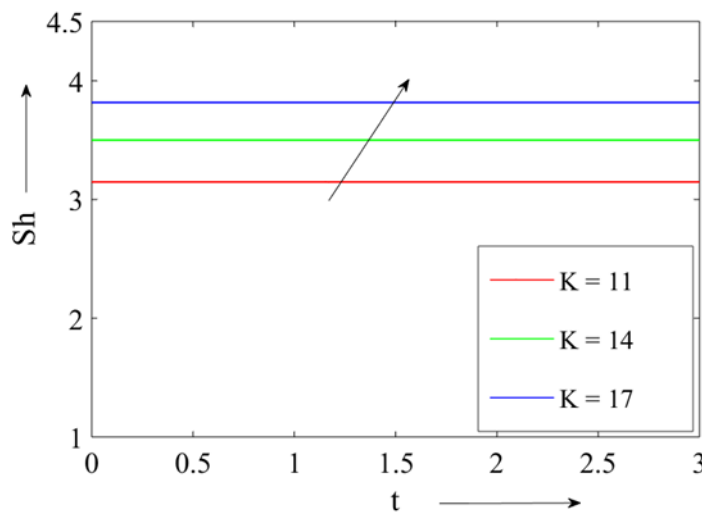


Figure 14. Sherwood number Sh vs. t for variations in K when $Sc=0.7$, $\varepsilon = 0.01$, $A=0.01$, $\omega = 2$.

Figure 12 illustrates that the rate of heat transfer Nu increases for the increasing effect of Dufour number Du or diffusion-thermo effect. But from Figure 13, it is seen that the rate of heat transfer decreases for high radiation. Figure 14 present the variation of Sherwood number against time t under chemical reaction parameter (K). It is seen in figure 14 that the mass transfer rate is enhanced under the chemical reaction effect, which indicates the fact that the rate of mass transfer increases for the consumption of species.

4. Conclusions

We have investigated MHD free convection from a semi-infinite vertical porous plate with a diffusion-thermo effect. We presented the results to illustrate the flow characteristics for the velocity, temperature, and concentration and the effects of the physical parameters of the flow. Therefore, we conclude that fluid velocity increases under the diffusion-thermo effect and thermal buoyancy force but decreases for increasing values of Hartmann number. The temperature of the fluid rises due to the Dufour effect but drops substantially under the radiation parameter effect. A significant downfall in fluid concentration is observed due to increased chemical reaction parameters and Schmidt number. Viscous drag at the plate increases under the diffusion-thermo effect, but it decreases for high radiation. The rate of heat transfer is enhanced with an increase in the Dufour effect, while a gradual decrement in this physical

quantity is noted for high radiation. The rate of mass transfer rises under the chemical reaction effect.

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Conflicts of Interest

The authors declare no conflict of interest.

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