

Computation of M -Polynomial and Topological Indices of Boron Kagome Lattice

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Abstract: In chemical graph theory, numerical encoding of chemical structure associated with topological indices is growing immensely. Prediction of the characteristics specified by the molecule's chemical structure is a salient feature of these topological indices. In this paper, we obtained the M -polynomial of the two-dimensional Boron Kagome Lattice. Some topological indices defined based on the degree of vertices can be computed gradually using the contemplated M -polynomial of this lattice. Further, we also provide the graphical representation of the M -polynomial and computed topological indices for the same.

Keywords: M -polynomial; Topological indices; Boron Kagome Lattice.

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1. Introduction

Topological indices, which Weiner in 1947 first started, is a logical process to convert the information of chemical compounds into some numbers. These indices provide many more chemical graph theory applications, especially in QSAR and QSPR studies. Topological indices can be categorized as degree-based, distance-based, eccentricity-based, spectrum-based, and so on. The degree-based topological indices are computed by degrees of the vertices of the chemical structures.

There are lots of degree-dependent topological indices introduced and studied [1-10]. Gutman and Trinajstić introduced two graph descriptors in 1972, named first Zagreb index and second Zagreb index. In 1982, A. T. Balaban introduced the Inverse sum-index of a graph. In 1987, S. Fajtlowicz introduced the Harmonic index. Bollobás and Erdős introduced the general randić index and the inverse randić index in 1998. In 2003, S. Nikoli introduced the Second modified Zagreb graph index. In 2010, D. Vukičević and M. Gasperov introduced the symmetric division deg index. In 2016 and 2017, V. R. Kulli introduced some new indices named first K-Banhatti index, second K-Banhatti index, modified first K-Banhatti index, modified second K-Banhatti index, and harmonic K-Banhatti index.

Graph polynomials encode the information of a graph and build up various algebraic methods to find out the hidden information of a graph. Several important graph algebraic polynomials have been introduced. Some of them are Hosoye polynomial [11], Matching polynomial [12], M -polynomial [13], and so on.

In 2015, the M -polynomial of a graph was introduced by E. Deutsch and S. Klavžar. To obtain the values of the topological indices, we can use another procedure by using derivatives or integral or both of the graph polynomials. M -polynomial is a polynomial that gives admirable results to determine degree-based topological indices.

The M -polynomial [13] of a graph G is illustrated as

$$M(G; x, y) = \sum_{\delta \leq a \leq b \leq \Delta} m_{ab}(G) x^a y^b$$

where $\delta = \min \{d_v | v \in V(G)\}$, $\Delta = \max \{d_v | v \in V(G)\}$ and $m_{ab}(G)$ is the number of edges $uv \in E(G)$ such that $(d_u, d_v) = (a, b)$.

Recently, many authors have studied the M -polynomials of several graph structures. In 2019, Raheem *et al.* [14] studied the M -polynomial of the two-dimensional three-layered single-walled titania nanotube lattice. In 2020, Khalaf *et al.* [15] obtained the book graph's M -polynomial and several topological indices. In 2020, Cancan *et al.* [16] introduced some topological indices via M -polynomial in the case of silicate networks. Some other important works can be found in [17-26].

In this current work, we study the M -polynomial of Boron Kagome Lattice and use it to derive some key degree-based topological indices of this lattice which is denoted by $BKL\{(m, n), x, y\}$. Here, a two-dimensional Boron Kagome Lattice (MgB_6) [27] has been considered. In the structure of Boron Kagome Lattice (MgB_6), between every pair of Boron Kagome layers, a triangular magnesium layer is sandwiched, which is shown in Figure 1(b). From the top view, Magnesium atoms are located at the centers of the hexagonal holes of the Boron Kagome layer. There is no direct connection between two Magnesium atoms; however, it is being surrounded by six boron atoms resulting in its degree-6. Whereas Boron atoms are of degree-4 as seen from the lattice structure.

The unique structures of this lattice promote electron transfer and stabilize the two-dimensional structure due to the formation of the covalent bond of the Boron Kagome layer and the ionic bond between Boron and Magnesium. In nanoelectronics, this material can be used as a super-thin electrode. The two-dimensional MgB_6 has superconductivity and hence can also be used as a superconductor besides MgB_2 . Another use of this material is hydrogen energy storage in nano energy applications.

In the structure of Boron Kagome Lattice, as shown below, the green-colored vertices represent the Magnesium atoms, and the yellow-colored vertices represent the Boron atoms of the lattice.

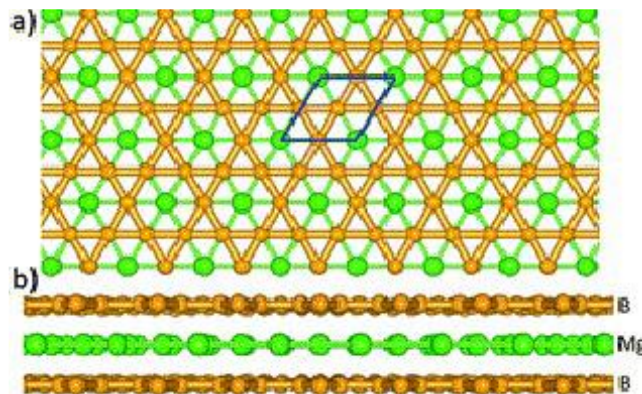


Figure 1. (a) Top and (b) side view of Boron Kagome Lattice.

For better understanding, we provide the structure of this lattice for some specific values of m and n as follows. In Figure 2, green-colored edges represent the connection between Magnesium and Boron atoms, whereas the yellow-colored edges for the connection

of two Boron atoms. In Figure 3, we represent several colors for distinguishing the edge partitions like grey colored edges for the connection between 2-degree and 4-degree vertices and also for 3-degree and 3-degree vertices, green color for 2-degree and 5-degree vertices, orange color for 3-degree and 4-degree vertices, pink color for 3-degree and 6-degree vertices, yellow color for 4-degree and 4-degree vertices, violet color for 4-degree and 5-degree vertices, blue color for 4-degree and 6-degree vertices, red color for 5-degree and 6-degree vertices and black color for 6-degree and 6-degree vertices.

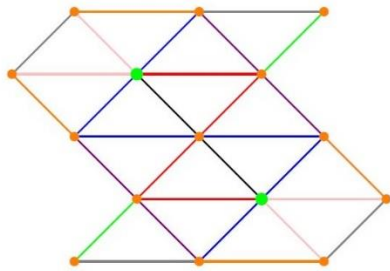


Figure 2. for $m = 1, n = 1$.

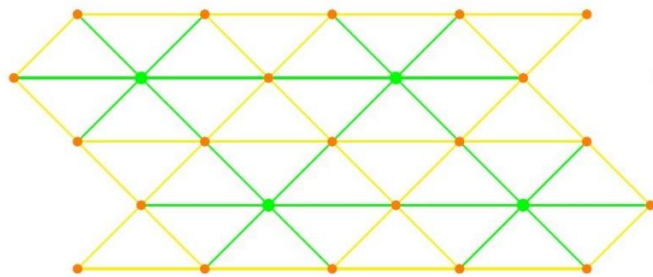


Figure 3. for $m = 1, n = 2$.

2. Some Topological Indices Derivations from M-Polynomial

Some familiar degree-based topological indices and their association with M-Polynomial are given in Table 1.

Table 1. Topological indices obtained from M-Polynomial where $M(G; x, y) = g(x, y)$.

Sr. no	Topological index	Derivation from $M(G; x, y)$
1	First Zagreb index	$(D_x + D_y)(g(x, y))_{x=y=1}$
2	Second Zagreb index	$D_x D_y (g(x, y))_{x=y=1}$
3	Second modified Zagreb index	$S_x S_y (g(x, y))_{x=y=1}$
4	Symmetric division deg index	$(D_x S_y + S_x D_y)(g(x, y))_{x=y=1}$
5	Harmonic index	$2S_x J(g(x, y))_{x=1}$
6	Inverse sum index	$S_x J D_x D_y (g(x, y))_{x=1}$
7	Inverse Randić index	$S_x^\alpha S_y^\alpha (g(x, y))_{x=y=1}$
8	General Randić index	$D_x^\alpha D_y^\alpha (g(x, y))_{x=y=1}$
9	First K-Banhatti index	$(D_x + D_y + 2D_x Q_{-2} J)(g(x, y))_{x=y=1}$
10	Second K-Banhatti index	$D_x Q_{-2} J(D_x + D_y)(g(x, y))_{x=1}$
11	Modified first K-Banhatti index	$S_x Q_{-2} J(L_x + L_y)(g(x, y))_{x=1}$
12	Modified second K-Banhatti index	$S_x Q_{-2} J(S_x + S_y)(g(x, y))_{x=1}$
13	Harmonic K-Banhatti index	$2S_x Q_{-2} J(L_x + L_y)(g(x, y))_{x=1}$

Where, $D_x = x \frac{\partial g(x,y)}{\partial x}$, $D_y = y \frac{\partial g(x,y)}{\partial y}$, $Jg(x, y) = g(x, x)$, $S_x = \int_0^x \frac{g(x,y)}{x} dx$, $S_y = \int_0^y \frac{g(x,y)}{y} dy$, $Q_\alpha g(x, y) = x^\alpha g(x, y)$, $L_x = g(x^2, y)$ and $L_y = g(x, y^2)$.

3. M-polynomial of Boron Kagome lattice.

Theorem 3.1. Let, $BKL_{(m,n)}$ be the graph of Boron Kagome Lattice. Then, the M-polynomial of $BKL_{(m,n)}$ is given by

$$\begin{aligned}
 M(BKL_{(m,n)}, x, y) &= 2x^2y^4 + 2x^2y^5 + 2x^3y^3 + 4mx^3y^4 + (2m + 2)x^3y^6 + (4n - 4)x^4y^4 \\
 &+ 4mx^4y^5 + (8m + 8n - 10)x^4y^6 + (6m - 2)x^5y^6 \\
 &+ (24mn - 20m - 10n + 8)x^6y^6.
 \end{aligned}$$

Proof. From the graph of Boron Kagome Lattice, we found five vertex partitions as follows,

$$\begin{aligned}
 V_2 &= \{v \in V(BKL_{(m,n)}) | d_v = 2\}, & V_3 &= \{v \in V(BKL_{(m,n)}) | d_v = 3\}, \\
 V_4 &= \{v \in V(BKL_{(m,n)}) | d_v = 4\}, & V_5 &= \{v \in V(BKL_{(m,n)}) | d_v = 5\}, \\
 V_6 &= \{v \in V(BKL_{(m,n)}) | d_v = 6\}.
 \end{aligned}$$

And we found ten edge partitions as follows,

$$\begin{aligned}
 E_{\{2,4\}} &= \{e = uv \in E(BKL_{(m,n)}) | d_u = 2, d_v = 4\}, \\
 E_{\{2,5\}} &= \{e = uv \in E(BKL_{(m,n)}) | d_u = 2, d_v = 5\}, \\
 E_{\{3,3\}} &= \{e = uv \in E(BKL_{(m,n)}) | d_u = 3, d_v = 3\}, \\
 E_{\{3,4\}} &= \{e = uv \in E(BKL_{(m,n)}) | d_u = 3, d_v = 4\}, \\
 E_{\{3,6\}} &= \{e = uv \in E(BKL_{(m,n)}) | d_u = 3, d_v = 6\}, \\
 E_{\{4,4\}} &= \{e = uv \in E(BKL_{(m,n)}) | d_u = 4, d_v = 4\}, \\
 E_{\{4,5\}} &= \{e = uv \in E(BKL_{(m,n)}) | d_u = 4, d_v = 5\}, \\
 E_{\{4,6\}} &= \{e = uv \in E(BKL_{(m,n)}) | d_u = 4, d_v = 6\}, \\
 E_{\{5,6\}} &= \{e = uv \in E(BKL_{(m,n)}) | d_u = 5, d_v = 6\}, \\
 E_{\{6,6\}} &= \{e = uv \in E(BKL_{(m,n)}) | d_u = 6, d_v = 6\}.
 \end{aligned}$$

From the graph of Boron Kagome Lattice $BKL_{(m,n)}$, we can scrutinize that $|E_{\{2,4\}}| = 2$, $|E_{\{2,5\}}| = 2$, $|E_{\{3,3\}}| = 2$, $|E_{\{3,4\}}| = 4m$, $|E_{\{3,6\}}| = 2m + 2$, $|E_{\{4,4\}}| = 4n - 4$, $|E_{\{4,5\}}| = 4m$, $|E_{\{4,6\}}| = 8m + 8n - 10$, $|E_{\{5,6\}}| = 6m - 2$, $|E_{\{6,6\}}| = 24mn - 20m - 10n + 8$.

Thus, by the definition of M -polynomial, we have

$$\begin{aligned}
 M(BKL_{(m,n)}; x, y) &= \sum_{i \leq j} m_{ij} (BKL_{(m,n)}) x^i y^j \\
 &= m_{24}(BKL_{(m,n)}) x^2 y^4 + m_{25}(BKL_{(m,n)}) x^2 y^5 + \\
 &m_{33}(BKL_{(m,n)}) x^3 y^3 + m_{34}(BKL_{(m,n)}) x^3 y^4 + m_{36}(BKL_{(m,n)}) x^3 y^6 + \\
 &m_{44}(BKL_{(m,n)}) x^4 y^4 + m_{45}(BKL_{(m,n)}) x^4 y^5 + m_{46}(BKL_{(m,n)}) x^4 y^6 + \\
 &m_{56}(BKL_{(m,n)}) x^5 y^6 + m_{66}(BKL_{(m,n)}) x^6 y^6 \\
 &= |E_{\{2,4\}}| x^2 y^4 + |E_{\{2,5\}}| x^2 y^5 + |E_{\{3,3\}}| x^3 y^3 + |E_{\{3,4\}}| x^3 y^4 + \\
 &|E_{\{3,6\}}| x^3 y^6 + |E_{\{4,4\}}| x^4 y^4 + |E_{\{4,5\}}| x^4 y^5 + |E_{\{4,6\}}| x^4 y^6 + |E_{\{5,6\}}| x^5 y^6 + |E_{\{6,6\}}| x^6 y^6 \\
 &= 2x^2y^4 + 2x^2y^5 + 2x^3y^3 + 4mx^3y^4 + (2m + 2)x^3y^6 + \\
 &(4n - 4)x^4y^4 + 4mx^4y^5 + (8m + 8n - 10)x^4y^6 + (6m - 2)x^5y^6 + (24mn - 20m - \\
 &10n + 8)x^6y^6. \blacksquare
 \end{aligned}$$

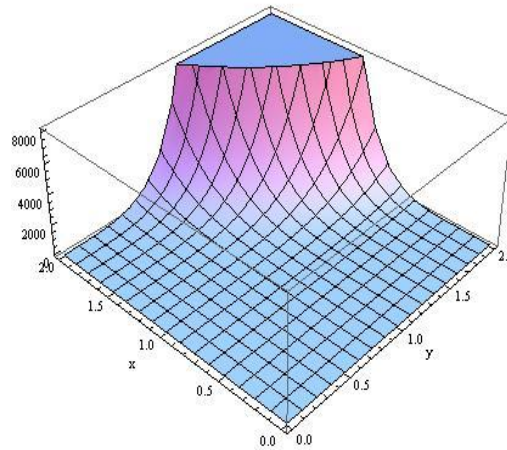


Figure 4. M-Polynomial of Boron Kagome Lattice.

With the help of this M - polynomial, we compute some topological indices.

Theorem 3.2. Let, $BKL_{(m,n)}$ be the graph of Boron Kagome Lattice and $M(BKL_{(m,n)}; x, y)$ be the M -polynomial of this lattice, then

1. $M_1(BKL_{(m,n)}) = 288mn - 12m - 8n - 2$
2. $M_2(BKL_{(m,n)}) = 864mn - 184m - 104n + 14$
3. $\bar{M}_2(BKL_{(m,n)}) = \frac{121}{180} + \frac{120mn+112m+55n-72}{180}$
4. $SDD(BKL_{(m,n)}) = \frac{74}{5} + \frac{720mn+166m+80n-191}{15}$
5. $H(BKL_{(m,n)}) = \frac{40}{21} + \frac{13860mn+6354m+3234n-5495}{3465}$
6. $I(BKL_{(m,n)}) = \frac{179}{21} + \frac{249480mn-16252m-9702n-32760}{3465}$
7. $RR_\alpha(BKL_{(m,n)}) = \left(\frac{1}{2^{3\alpha-1}} + \frac{1}{2^{\alpha-1}5^\alpha} + \frac{2}{3^{2\alpha}} \right) + \frac{m}{4^{\alpha-1}3^\alpha} + \frac{m+1}{2^{\alpha-1}3^{2\alpha}} + \frac{n-1}{2^{4\alpha-2}} + \frac{m}{4^{\alpha-1}5^\alpha} + \frac{4m+4n-5}{2^{3\alpha-1}3^\alpha} + \frac{3m-1}{2^{\alpha-1}3^\alpha 5^\alpha} + \frac{12mn-10m-5n+4}{2^{2\alpha-1}3^{2\alpha}}$
8. $R_\alpha(BKL_{(m,n)}) = (2^{3\alpha+1} + 2^{\alpha+1} \cdot 5^\alpha + 2 \cdot 3^{2\alpha}) + 4m \cdot 2^{2\alpha} \cdot 3^\alpha + 2(m+1) \cdot 2^\alpha \cdot 3^{2\alpha} + (n-1) \cdot 2^{2+4\alpha} + m \cdot 2^{2+2\alpha} \cdot 5^\alpha + (4m+4n-5) \cdot 2^{3\alpha+1} \cdot 3^\alpha + (3m-1) \cdot 2^{\alpha+1} \cdot 3^\alpha \cdot 5^\alpha + (12mn-10m-5n+4) \cdot 2^{1+2\alpha} \cdot 3^{2\alpha}$
9. $B_1(BKL_{(m,n)}) = 768mn - 52m - 32n - 6$
10. $B_2(BKL_{(m,n)}) = 2880mn - 648m - 368n + 62$
11. $\bar{B}_1(BKL_{(m,n)}) = \frac{25}{21} + \frac{540540mn+281428m+141999n-147741}{180180}$
12. $\bar{B}_2(BKL_{(m,n)}) = \frac{17}{24} + \frac{60480mn+65100m+31500n-18607}{75600}$

$$13. H_b(BKL_{(m,n)}) = \frac{50}{21} + \frac{540540mn+281428m+141999n-147741}{90090}$$

Proof. From the definition of $D_x, D_y, Jg(x, y), S_x, S_y, Q_\alpha(g(x, y)), L_x, L_y$ we have,

$$(D_x + D_y)(g(x, y)) = 12x^2y^4 + 14x^2y^5 + 12x^3y^3 + 28mx^3y^4 + (18m + 18)x^3y^6 + (32n - 32)x^4y^4 + 36mx^4y^5 + (80m + 80n - 100)x^4y^6 + (66m - 22)x^5y^6 + (288mn - 240m - 120n + 96)x^6y^6,$$

$$(S_x + S_y)(g(x, y)) = \left(\frac{3}{2}\right) \cdot x^2y^4 + \left(\frac{7}{5}\right) \cdot x^2y^5 + \left(\frac{4}{3}\right) \cdot x^3y^3 + \left(\frac{7m}{3}\right) \cdot x^3y^4 + (m + 1) \cdot x^3y^6 + (2n - 2)x^4y^4 + \left(\frac{9m}{5}\right) \cdot x^4y^5 + \left(\frac{20m+20n-25}{6}\right) \cdot x^4y^6 + \left(\frac{33m-11}{15}\right) \cdot x^5y^6 + \left(\frac{24mn-20m-10n+8}{3}\right) \cdot x^6y^6,$$

$$(L_x + L_y)(g(x, y)) = 2x^4y^4 + 2x^4y^5 + 2x^6y^3 + 4m \cdot x^6y^4 + (2m + 2) \cdot x^6y^6 + (4n - 4) \cdot x^8y^4 + 4m \cdot x^8y^5 + (8m + 8n - 10) \cdot x^8y^6 + (6m - 2) \cdot x^{10}y^6 + (24mn - 20m - 10n + 8) \cdot x^{12}y^6 + 2x^2y^8 + 2x^2y^{10} + 2x^3y^6 + 4m \cdot x^3y^8 + (2m + 2) \cdot x^3y^{12} + (4n - 4) \cdot x^4y^8 + 4m \cdot x^4y^{10} + (8m + 8n - 10) \cdot x^4y^{12} + (6m - 2) \cdot x^5y^{12} + (24mn - 20m - 10n + 8) \cdot x^6y^{12},$$

$$(D_x D_y)(g(x, y)) = 16x^2y^4 + 20x^2y^5 + 18x^3y^3 + 48m \cdot x^3y^4 + (36m + 36) \cdot x^3y^6 + (64n - 64) \cdot x^4y^4 + 80m \cdot x^4y^5 + (192m + 192n - 240) \cdot x^4y^6 + (180m - 60) \cdot x^5y^6 + (864mn - 720m - 360n + 288) \cdot x^6y^6,$$

$$(S_x S_y)(g(x, y)) = \left(\frac{1}{4}\right) \cdot x^2y^4 + \left(\frac{1}{5}\right) \cdot x^2y^5 + \left(\frac{2}{9}\right) \cdot x^3y^3 + \left(\frac{m}{3}\right) \cdot x^3y^4 + \left(\frac{m+1}{9}\right) \cdot x^3y^6 + \left(\frac{n-1}{4}\right) \cdot x^4y^4 + \left(\frac{m}{5}\right) \cdot x^4y^5 + \left(\frac{4m+4n-5}{12}\right) \cdot x^4y^6 + \left(\frac{3m-1}{15}\right) \cdot x^5y^6 + \left(\frac{12mn-10m-5n+4}{18}\right) \cdot x^6y^6,$$

$$(D_x S_y)(g(x, y)) = x^2y^4 + \left(\frac{4}{5}\right) \cdot x^2y^5 + 2x^3y^3 + 3m \cdot x^3y^4 + (m + 1) \cdot x^3y^6 + 4(n - 1) \cdot x^4y^4 + \left(\frac{16m}{5}\right) \cdot x^4y^5 + \left(\frac{16m+16n-20}{3}\right) \cdot x^4y^6 + \left(\frac{15m-5}{3}\right) \cdot x^5y^6 + (24mn - 20m - 10n + 8) \cdot x^6y^6,$$

$$(S_x D_y)(g(x, y)) = 4x^2y^4 + 5x^2y^5 + 2x^3y^3 + \left(\frac{16m}{3}\right) \cdot x^3y^4 + (4m + 4) \cdot x^3y^6 + 4(n - 1) \cdot x^4y^4 + 5mx^4y^5 + (12m + 12n - 15) \cdot x^4y^6 + \left(\frac{36m-12}{5}\right) \cdot x^5y^6 + (24mn - 20m - 10n + 8) \cdot x^6y^6,$$

$$J(D_x + D_y)(g(x, y)) = 24x^6 + (14 + 28m) \cdot x^7 + (32n - 32) \cdot x^8 + (18 + 54m) \cdot x^9 + (80m + 80n - 100) \cdot x^{10} + (66m - 22) \cdot x^{11} + (288mn - 240m - 120n + 96) \cdot x^{12},$$

$$J(S_x + S_y)(g(x, y)) = \left(\frac{17}{6}\right) \cdot x^6 + \left(\frac{21+35m}{15}\right) \cdot x^7 + (2n - 2) \cdot x^8 + \left(\frac{42m+15}{15}\right) \cdot x^9 + \left(\frac{20m+20n-25}{6}\right) \cdot x^{10} + \left(\frac{33m-11}{15}\right) \cdot x^{11} + \left(\frac{24mn-20m-10n+8}{3}\right) x^{12},$$

$$J(D_x D_y)(g(x, y)) = 34x^6 + (20 + 48m) \cdot x^7 + (64n - 64) \cdot x^8 + (36 + 116m) \cdot x^9 + (192m + 192n - 240) \cdot x^{10} + (180m - 60) \cdot x^{11} + (864mn - 720m - 360n + 288) \cdot x^{12},$$

$$S_x J D_x D_y(g(x, y)) = \left(\frac{17}{3}\right) \cdot x^6 + \left(\frac{20+48m}{7}\right) \cdot x^7 + (8n - 8) \cdot x^8 + \left(\frac{36+116m}{9}\right) \cdot x^9 + \left(\frac{96m+96n-120}{5}\right) \cdot x^{10} + \left(\frac{18m-60}{11}\right) \cdot x^{11} + (72mn - 60m - 30n + 24) \cdot x^{12},$$

$$(S_x^\alpha S_y^\alpha)(g(x, y)) = \left(\frac{1}{2^{3\alpha-1}}\right) \cdot x^2 y^4 + \left(\frac{1}{2^{\alpha-1.5\alpha}}\right) \cdot x^2 y^5 + \left(\frac{2}{3^{2\alpha}}\right) \cdot x^3 y^3 + \left(\frac{m}{4^{\alpha-1.3\alpha}}\right) \cdot x^3 y^4 + \left(\frac{m+1}{2^{\alpha-1.3^{2\alpha}}}\right) \cdot x^3 y^6 + \left(\frac{n-1}{2^{4\alpha-2}}\right) \cdot x^4 y^4 + \left(\frac{m}{4^{\alpha-1.5\alpha}}\right) \cdot x^4 y^5 + \left(\frac{4m+4n-5}{2^{3\alpha-1.3\alpha}}\right) \cdot x^4 y^6 + \left(\frac{3m-1}{2^{\alpha-1.3\alpha.5\alpha}}\right) \cdot x^5 y^6 + \left(\frac{12mn-10m-5n+4}{2^{2\alpha-1.3^{2\alpha}}}\right) \cdot x^6 y^6,$$

$$(D_x^\alpha D_y^\alpha)(g(x, y)) = 2^{3\alpha+1} x^2 y^4 + 2^{\alpha+1} \cdot 5^\alpha x^2 y^5 + 2 \cdot 3^{2\alpha} x^3 y^3 + 4m \cdot 2^{2\alpha} \cdot 3^\alpha x^3 y^4 + 2(m + 1) \cdot 2^\alpha \cdot 3^{2\alpha} x^3 y^6 + (n - 1) \cdot 2^{2+4\alpha} x^4 y^4 + m \cdot 2^{2+2\alpha} \cdot 5^\alpha x^4 y^5 + (4m + 4n - 5) \cdot 2^{3\alpha+1} \cdot 3^\alpha x^4 y^6 + (3m - 1) \cdot 2^{\alpha+1} \cdot 3^\alpha \cdot 5^\alpha x^5 y^6 + (12mn - 10m - 5n + 4) \cdot 2^{1+2\alpha} \cdot 3^{2\alpha} x^6 y^6,$$

$$D_x Q_{-2} J(g(x, y)) = 16x^4 + (10 + 20m) \cdot x^5 + (24n - 24) \cdot x^6 + (14 + 42m) \cdot x^7 + (64m + 64n - 80) \cdot x^8 + (54m - 18) \cdot x^9 + (240mn - 200m - 100n + 80) \cdot x^{10},$$

$$Q_{-2} J(D_x + D_y)(g(x, y)) = 24x^4 + (14 + 28m) \cdot x^5 + (32n - 32) \cdot x^6 + (18 + 54m) \cdot x^7 + (80m + 80n - 100) \cdot x^8 + (66m - 22) \cdot x^9 + (288mn - 240m - 120n + 96) \cdot x^{10},$$

$$D_x Q_{-2} J(D_x + D_y)(g(x, y)) = 96x^4 + (70 + 140m) \cdot x^5 + (192n - 192) \cdot x^6 + (378 + 126m) \cdot x^7 + (640m + 640n - 800) \cdot x^8 + (594m - 198) \cdot x^9 + (2880mn - 2400m - 1200n + 960) \cdot x^{10},$$

$$Q_{-2} J(L_x + L_y)(g(x, y)) = 2x^6 + 6x^7 + (2 + 4m) \cdot x^8 + 4m \cdot x^9 + (2m + 8n - 4) \cdot x^{10} + 4m \cdot x^{11} + (12m + 8n - 10) \cdot x^{12} + (2m + 2) \cdot x^{13} + (14m + 8n - 12) \cdot x^{14} + (6m - 2) \cdot x^{15} + (480mn - 40m - 20n + 16) \cdot x^{16},$$

$$S_x Q_{-2} J(L_x + L_y)(g(x, y)) = \left(\frac{1}{3}\right) \cdot x^6 + \left(\frac{6}{7}\right) \cdot x^7 + \left(\frac{1+2m}{4}\right) \cdot x^8 + \left(\frac{4m}{9}\right) \cdot x^9 + \left(\frac{m+4n-2}{5}\right) \cdot x^{10} + \left(\frac{4m}{11}\right) \cdot x^{11} + \left(\frac{6m+4n-5}{6}\right) \cdot x^{12} + \left(\frac{2m+2}{13}\right) \cdot x^{13} + \left(\frac{7m+4n-6}{7}\right) \cdot x^{14} + \left(\frac{6m-2}{15}\right) \cdot x^{15} + \left(\frac{12mn-10m-5n+4}{4}\right) \cdot x^{16},$$

$$Q_{-2} J(S_x + S_y)(g(x, y)) = \left(\frac{17}{6}\right) \cdot x^4 + \left(\frac{21+35m}{15}\right) \cdot x^5 + (2n - 2) \cdot x^6 + \left(\frac{42m+15}{15}\right) \cdot x^7 + \left(\frac{20m+20n-25}{6}\right) \cdot x^8 + \left(\frac{33m-11}{15}\right) \cdot x^9 + \left(\frac{24mn-20m-10n+8}{3}\right) \cdot x^{10},$$

Now, using these above data and Table 1, we have the following indices:

1. First Zagreb index

$$M_1(BKL_{(m,n)}) = (D_x + D_y)(g(x, y))_{x=y=1} = 288mn - 12m - 8n - 2$$

2. Second Zagreb index

$$M_2(BKL_{(m,n)}) = (D_x D_y)(g(x, y))_{x=y=1} = 864mn - 184m - 104n + 14$$

3. Second modified Zagreb index

$$\bar{M}_2(BKL_{(m,n)}) = (S_x S_y)(g(x, y))_{x=y=1} = \frac{121}{180} + \frac{120mn + 112m + 55n - 72}{180}$$

4. Symmetric division deg index

$$SDD(BKL_{(m,n)}) = (D_x S_y + S_x D_y)(g(x, y))_{x=y=1} = \frac{74}{5} + \frac{720mn + 166m + 80n - 191}{15}$$

5. Harmonic index

$$H(BKL_{(m,n)}) = 2S_x J(g(x, y))_{x=1} = \frac{40}{21} + \frac{13860mn + 6354m + 3234n - 5495}{3465}$$

6. Inverse sum index

$$I(BKL_{(m,n)}) = S_x J D_x D_y (g(x, y))_{x=1} \\ = \frac{179}{21} + \frac{249480mn - 16252m - 9702n - 32760}{3465}$$

7. Inverse Randić index

$$RR_\alpha(BKL_{(m,n)}) = (S_x^\alpha S_y^\alpha)(g(x, y))_{x=y=1} = \left(\frac{1}{2^{3\alpha-1}} + \frac{1}{2^{\alpha-1} 5^\alpha} + \frac{2}{3^{2\alpha}} \right) + \frac{m}{4^{\alpha-1} 3^\alpha} + \\ \frac{m+1}{2^{\alpha-1} 3^{2\alpha}} + \frac{n-1}{2^{4\alpha-2}} + \frac{m}{4^{\alpha-1} 5^\alpha} + \frac{4m+4n-5}{2^{3\alpha-1} 3^\alpha} + \frac{3m-1}{2^{\alpha-1} 3^\alpha 5^\alpha} + \frac{12mn-10m-5n+4}{2^{2\alpha-1} 3^{2\alpha}}$$

8. General Randić index

$$R_\alpha(BKL_{(m,n)}) = (D_x^\alpha D_y^\alpha)(g(x, y))_{x=y=1} \\ = (2^{3\alpha+1} + 2^{\alpha+1} \cdot 5^\alpha + 2 \cdot 3^{2\alpha}) + 4m \cdot 2^{2\alpha} \cdot 3^\alpha + 2(m+1) \cdot 2^\alpha \cdot 3^{2\alpha} \\ + (n-1) \cdot 2^{2+4\alpha} + m \cdot 2^{2+2\alpha} \cdot 5^\alpha + (4m+4n-5) \cdot 2^{3\alpha+1} \cdot 3^\alpha \\ + (3m-1) \cdot 2^{\alpha+1} \cdot 3^\alpha \cdot 5^\alpha + (12mn - 10m - 5n + 4) \cdot 2^{1+2\alpha} \cdot 3^{2\alpha}$$

9. First K Banhatti index

$$B_1(BKL_{(m,n)}) = (D_x + D_y + 2D_x Q_{-2} J)(g(x, y))_{x=y=1} = 768mn - 52m - 32n - 6$$

10. Second K Banhatti index

$$B_2(BKL_{(m,n)}) = (D_x Q_{-2} J(D_x + D_y))(g(x, y))_{x=1} = 2880mn - 648m - 368n + 62$$

11. Modified First K Banhatti index

$$\bar{B}_1(BKL_{(m,n)}) = S_x Q_{-2} J(L_x + L_y)(g(x, y))_{x=1} \\ = \frac{25}{21} + \frac{540540mn + 281428m + 141999n - 147741}{180180}$$

12. Modified Second K Banhatti index

$$\begin{aligned} \bar{B}_2(BKL_{(m,n)}) &= Q_{-2}J(S_x + S_y)(g(x, y))_{x=1} \\ &= \frac{17}{24} + \frac{60480mn + 65100m + 31500n - 18607}{75600} \end{aligned}$$

13. Harmonic K Banhatti index

$$\begin{aligned} H_b(BKL_{(m,n)}) &= 2S_x Q_{-2}J(L_x + L_y)(g(x, y))_{x=1} \\ &= \frac{50}{21} + \frac{540540mn + 281428m + 141999n - 147741}{90090} \end{aligned}$$

It will be more interesting to represent these indices by graphical notations. Therefore, we provide the visualization of the three-dimensional graphical representations for each of these indices.

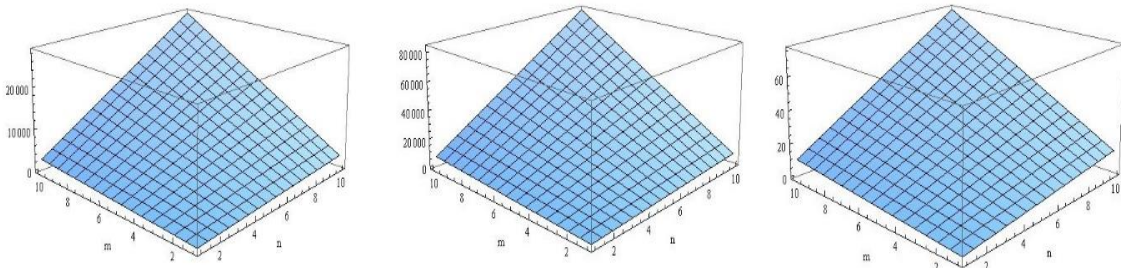


Figure 5. First Zagreb, second Zagreb, and Second modified Zagreb indices of Boron Kagome Lattice.

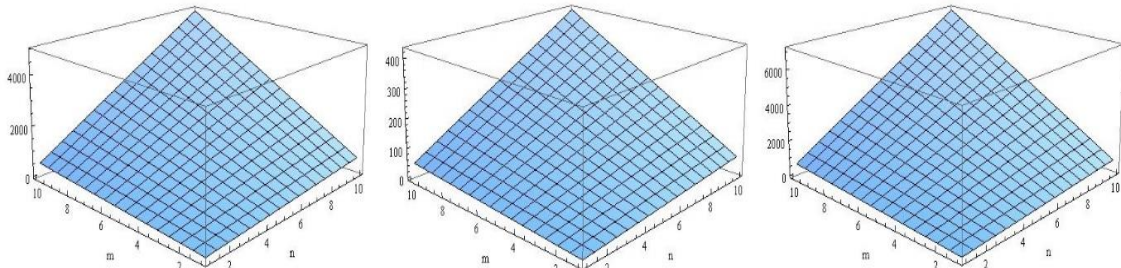


Figure 6. Symmetric division deg index, Harmonic index, and Inverse sum index of Boron Kagome Lattice.

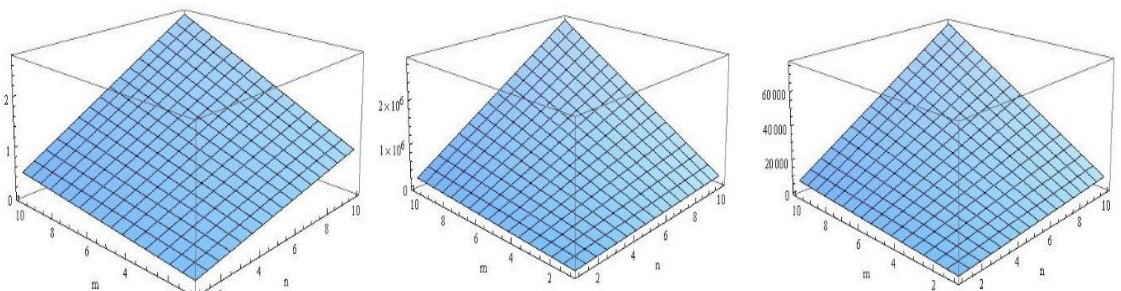


Figure 7. Inverse Randić index, General Randić index, and First K-Banhatti index of Boron Kagome Lattice.

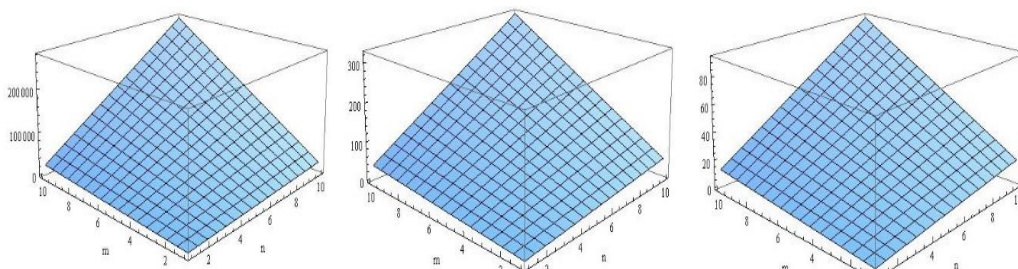


Figure 8. Second K-Banhatti index, Modified First K-Banhatti index, and Modified Second K-Banhatti index of Boron Kagome Lattice.

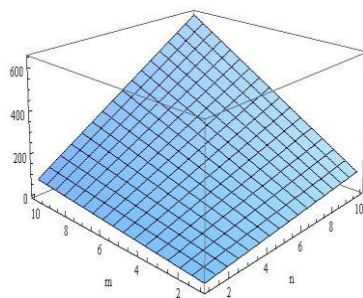


Figure 9. Harmonic K-Banhatti index of Boron Kagome Lattice.

4. Conclusions

In this paper, we obtain the M -polynomial of two-dimensional Boron Kagome Lattice, and with the help of this polynomial, we have computed some topological indices of this lattice. We also provide the three-dimensional graphical representation of the M -polynomial and the degree-based topological indices calculated. These graphical representations allow us to understand results against parameters. All indices are increases with respect to the graph parameter increases. Among the computed topological indices, the second K-Banhatti index grows fast while the inverse randić index grows slowly. For future directions, we can use the same technique to obtain the M -polynomial and topological indices of other chemical structures.

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Conflicts of Interest

The authors declare no conflict of interest.

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