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Uniform Flow of Viscous Fluid Past a Porous Sphere Saturated with Micro Polar Fluid

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Abstract: The analytical investigation of a uniform flow of a viscous fluid past a spherical ball filled with porous medium saturated by Micropolar fluid is considered. Velocity function is defined in the form of stream function. The flowing shape is obtained for the outer and inner regions of the sphere. The effects of physical parameters like porosity and micro polarity parameters on the flow and on the drag on the sphere are shown in graphs. It is observed that when the porosity parameter is between 0.05 to 0.2, we find another concentric fluid sphere enclosing the porous sphere. Below and above this range, the fluid sphere disappears.

Keywords: micropolar fluid; permeable sphere; non-stick and hyper stick conditions.

Nomenclature: $\bar{q}$: Velocity Vector $\bar{v}$: Micro-Rotation Vector- p : Pressure
ρ : Density of Micro-Polar fluid α, β, γ : Material constants for gyro viscosity coefficients
μ, k, λ : Material constants for viscosity coefficients
j : Micro-gyration

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1. Introduction

Many investigators attracted classical problems of flow past permeable bodies since it has theoretical and experimental interest, and it has so many applications in the industry field. Leonov [1], in his paper, calculated the stream function formula for external and internal flows for viscous fluid in the case of a porous sphere. Using Stokes approximation, Varma and Gour [2] investigated the oscillatory flow of a viscous incompressible fluid past a fixed porous sphere. They have determined the stream function and velocity components in terms of porosity parameters for the flow outside and inside the sphere. Padmavathi[3] *et al*. attained a common method for calculating non-axi-symmetric flow both for inner and outer regions of the permeable spherical boundary when there is a Stokes flow of viscous liquid past the body. Qin Yu and Kalony [4] have described the creeping flow past a porous spherical shell when a shell is thick enough to be assumed as a porous medium. Jones [5] assumed the flow of a viscous fluid around and through a porous spherical shell. He discussed diverse boundary conditions and proposed boundary conditions relevant to curved surfaces. Bhatt and Nirmal Sacheti [6] have examined the slow motion of a Newtonian fluid past a porous spherical shell. They

assumed that the Navier- Stokes equations govern the flow outside the porous spherical region and within the core region while using the Brinkman model for the flow inside the porous region.

Joseph and Tao [7] analyzed the flow of an incompressible viscous fluid past a porous spherical particle by using Darcy's law in the porous region and no-slip condition for the tangential velocity on the surface of a sphere. Dennis *et al*. [8] calculated the numerical steady of rotation of a sphere in a viscous fluid using Reynolds numbers. They showed that the torque exerted by the fluid on the sphere is found to be in good agreement with the experimental and theoretical results at low Reynolds numbers. Ramana Murthy and Aparna [9, 10] calculated a formula for drag for uniform and oscillatory flows for a couple of stress fluids past a permeable sphere and studied the permeability parameter. Using an impulse applied to an immersed sphere, Felderhof [11] examined the transient flow for a viscous incompressible fluid. Aparna *et al*. [12] derived a formula for drag in the case of micropolar fluid for flow past sphere. The study emphasized the revolving motion of micropolar fluid in the sphere case by Aparna *et al*. [13]. Webster [14] derived a finite difference method to solve various fluid flow problems. Casanellas and Ortin [15] studied the Laminar Oscillatory flow problem. Jayalakshmamma *et al*. [16] studied the steady flow of an incompressible micropolar fluid past an impervious sphere using numerical techniques. Ramalakshmi and Pankaj Shukla [17] examined the flow past a porous sphere embedded in a micropolar fluid using numerical and analytical techniques. Ashmawy obtained a simple formula for drag for a couple of stress fluids in the case of the sphere [18]. Aparna *et al*., in their study, obtained the formula for swirl for rotary flow in case of couple stress fluid [19]. Mishra and Gupta calculated drag for micropolar fluid in the case of the composite sphere [20]. Recently Khanukaeva[21] studied fluid flow problems on spherical geometry. Krishna prasad *et al*.[22], Aparna [23], Krishna prasad *et al*.[24], Pankaj shukla *et al*. [25], Umadevi *et al*. [26], Asia Yasmin *et al*.[27], Noraihan Afiqah Rawi *et al*. [28] were studied fluid flow problems on the sphere and stretching sheets. Basha *et al*. [29], Muhammad Ashraf *et al*. [30], Jared Penney *et al*. [31], Krishna prasad *et al*. [32] were studied problems on the cylinder, El-Sayed Ibrah Zhe wang [33] reported about fluid flow problems on the prolate spheroid. Radha *et al*. [34], Yulia O.Koroleva *et al*. [35] studied fluid flow problems on the arbitrary shape and axially symmetric cells.

2. Modelling of the Problem

This research work considers the uniform flow of a viscous fluid past a fixed porous sphere of radius *a*. The porous region within the sphere is saturated with incompressible micro-polar fluid. The equations of motion under Stokesian assumption for micro-polar fluid in steady-state in the porous medium for the inside sphere are given by

$$\nabla.\bar{q}_i = 0 \tag{1}$$

$$0 = -\nabla p_i + k\nabla\times\bar{\upsilon} - (\mu + k)\nabla\times\nabla\times\bar{q}_i - \frac{\mu}{k^*}\bar{q}_i \tag{2}$$

$$0 = -2k\bar{v} + k\nabla\times\bar{q}_i - \gamma\nabla\times\nabla\times\bar{v} + (\alpha+\beta+\gamma)\nabla(\nabla\cdot\bar{v}) \tag{3}$$

For outside region, the equations of motion for viscous fluid under the Stokes assumptions are given by

$$\nabla\cdot\bar{q}_e = 0 \tag{4}$$

$$0 = -\nabla p_e - \mu \nabla \times (\nabla \times \bar{q}_e) \tag{5}$$

where $\bar{q} = \begin{cases} q_i & if \quad R < a \\ q_e & if \quad R \geq a \end{cases}$, $p = \begin{cases} p_i & if \quad R < a \\ p_e & if \quad R \geq a \end{cases}$

Here λ, μ, k are dimensions' viscosity coefficients and gyro viscosity coefficients of dimensions MLT^{-1} and κ^* coefficient of permeability. These coefficients are subject to the inequalities given below.

$$k \geq 0,\ 2\mu + k \geq 0,\ 3\lambda + 2\mu + k \geq 0$$
$$\gamma \geq 0,\ -\gamma \leq \beta \leq \gamma,\ 3\alpha + \beta + \gamma \geq 0$$

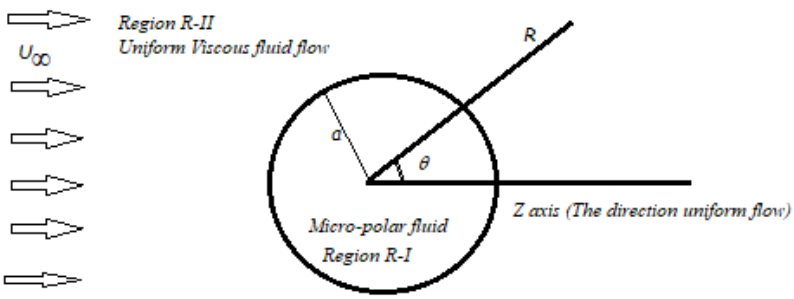


Figure 1. Uniform viscous fluid flow past a Porous sphere saturated with micro-polar fluid.

To satisfy incompressibility condition (1) or (4), we assume that velocity vector is expressed in terms of Stream function and micro rotation vectors as

$$\bar{q} = \nabla \times \left(\frac{\Psi \bar{e}_\phi}{h_3} \right) = U\,\bar{e}_r + V\,\bar{e}_\theta \tag{6}$$

and $\bar{\upsilon} = \frac{\mathscr{E}}{h_3} \bar{e}_\phi$ (7)

In view of equation (6) , the velocity components are

$$U = \frac{1}{R^2 \sin\theta} \frac{\partial \Psi}{\partial \theta},\ V = -\frac{1}{R \sin\theta} \frac{\partial \Psi}{\partial R}$$

Taking curl to equation (2), we get,

$$E_0^2 \left(E_0^2 - \frac{\mu}{(\mu + \kappa) k^*}\right) \Psi = -c E_0^2 \mathscr{E}$$

We note that equation (3) leads to

$$(E_0^2 - 2k/\gamma)\mathscr{E} = (k/\gamma) E_0^2 \Psi$$

where $E_0{}^2 = \frac{\partial^2}{\partial R^2} + \left(\frac{1 - x^2}{R^2} \right) \frac{\partial^2}{\partial x^2}$, and $x = \cos\theta$

The non-dimensional scheme is introduced as follows. On LHS, the capital letters are physical quantities, and the quantities on RHS are the corresponding non-dimensional quantities in small italic font.

$$R = ar,\ \Psi = U_\infty a^2\psi,\ P = \mu U_\infty a\ p,\ \mathcal{C} = U_\infty C\ , s = \frac{ka^2}{\gamma}, c = \frac{k}{\mu + k}, \mathrm{E}^2 = \mathrm{E}^2{}_0 a^2, Da = k^* a^2$$

The equations for the non-dimensional stream function for internal flow after eliminating C is given by

$$E^2(E^2 - \lambda_1^2)(E^2 - \lambda_2^2)\psi_i = 0 \tag{8}$$

where $\lambda_1^2 + \lambda_2^2 = \frac{(1-c)}{Da} + (2-c)s$ and $\lambda_1^2\,\lambda_2^2 = \frac{2s(1-c)}{Da}$

The equation for stream function for external flow after eliminating pressure from equations (4) and (5) is given by

$$E^4\psi_e = 0 \tag{9}$$

3. Solution of the Problem

The equation for velocity in terms of Stream function ψ for the inner region and outer region is given by equation (8).

By the method of separation of variables, we see that the solutions that are regular far away from the body and that are bounded are given by

$$\psi_e = \left(r^2 + \frac{a_1}{r} + b_1 r\right)G_2(x) \tag{10}$$

$$\psi_i = \left[a_2 r^2 + b_2\sqrt{r}\,I_{3/2}(\lambda_1 r) + c_2\sqrt{r}\,I_{3/2}(\lambda_2 r)\right]G_2(x) \tag{11}$$

where $G_2(x) = \frac{1}{2}(1-x^2)$ is a Gegenbauer's polynomial of degree 2 and $x = \cos\theta$, $I_{3/2}$ (x) is modified Bessel function of the second kind. The micro-rotation component C for internal flows reduces to

$$2csC = -E^2\left(E^2\psi_i - \left(\frac{1-c}{Da} - cs\right)\psi_i\right) = -E^2(E^2 - \lambda_1^2 - \lambda_2^2 + 2s)\psi_i$$

This implies that

$$2c\,s\,C = \left\{b_2\,\lambda_1^2\sqrt{r}\,I_{3/2}(\lambda_1 r)\left(\lambda_2^2 - 2s\right) + c_2\lambda_2^2\sqrt{r}\,I_{3/2}(\lambda_2 r)\left(\lambda_1^2 - 2s\right)\right\} \tag{12}$$

Applying the above boundary conditions, from (iii), we get

$$1 + a_1 + b_1 = a_2 + b_2 I_{3/2}(\lambda_1) + c_2 I_{3/2}(\lambda_2) \tag{13}$$

From (iv), we get:

$$2 - a_1 + b_1 = 0 \tag{14}$$

$$2a_2 - b_2 I_{3/2}(\lambda_1)\Delta(\lambda_1) - c_2 I_{3/2}(\lambda_2)\Delta(\lambda_2) = 0 \tag{15}$$

where $\Delta(x) = 1 - \frac{xI_{1/2}(x)}{I_{3/2}(x)} = 1 + \frac{x^2}{1 - x\coth x}$

From (v), we get;

$$b_2\,\lambda_1^2\,I_{3/2}(\lambda_1)\left(\lambda_2^2 - 2s\right) + c_2\lambda_2^2\,I_{3/2}(\lambda_2)\left(\lambda_1^2 - 2s\right) = 0 \tag{16}$$

From equation (2),

$$\nabla p_i = \frac{(\mu+k)U_\infty}{a^2}\{\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}[cC+E^2\psi_i-\frac{1-c}{Da}\psi_i]\bar{e}_r-\frac{1}{r\sin\theta}\frac{\partial}{\partial r}[cC+E^2\psi_i-\frac{1-c}{Da}\psi_i]\bar{e}_\theta\} \tag{17}$$

We notice that

$$cC+E^2\psi_i-\frac{1-c}{Da}\psi=-\frac{1}{2s}(E^2-\lambda_1^2)(E^2-\lambda_2^2)\psi_i=-a_2\frac{\lambda_1^2\lambda_2^2}{2s}r^2\frac{1}{2}\sin^2\theta \tag{18}$$

Introducing (18) into (17), we get the internal pressure as

$$P_i=-\frac{(\mu+k)}{a^2}a_2\frac{\lambda_1^2\lambda_2^2}{2s}r\cos\theta \tag{19}$$

From equation (5), we get

$$\nabla p_e=-\frac{\mu U_\infty}{a^2}\nabla\times(\frac{E^2\psi}{h_3})=\frac{\mu U_\infty}{a^2}\nabla\times(\frac{b_1}{r^2}\sin\theta\,\bar{e}_\phi)=\frac{\mu U_\infty b_1}{a^2}\{\frac{2\cos\theta}{r^3}\bar{e}_r+\frac{\sin\theta}{r^2}e_\theta\}$$

Integrating this we get;

$$P_e=\frac{\mu U_\infty}{a^2}b_1\frac{\cos\theta}{r} \tag{20}$$

Now using the condition of continuity of pressure on r=1 gives: $p_e=p_i$

$$\Rightarrow a_2\lambda_1^2\lambda_2^2=2s(1-c)b_1 \tag{21}$$

Hence finally, we get the following system of equations for a_1, b_1, a_2, b_2, c_2 :

$$a_1+b_1-a_2-b'_2-c'_2=-1$$

$$a_1-b_1=2$$

$$2a_2-b'_2\,\Delta(\lambda_1)-c'_2\,\Delta(\lambda_2)=0$$

$$b'_2\,\lambda_1^2\left(\lambda_2^2-2s\right)+c'_2\,\lambda_2^2\left(\lambda_1^2-2s\right)=0$$

and $a_2\lambda_1^2\lambda_2^2=2s(1-c)b_1$

where $b_2'=b_2I_{1.5}(\lambda_1)$ *and* $c_2'=c_2I_{1.5}(\lambda_2)$

By solving the Solving above equations, we get

$$c_2'\left\{1-\frac{\lambda_2^2(\lambda_1^2-2s)}{\lambda_1^2(\lambda_2^2-2s)}+\frac{1}{2}\left(1-\frac{\lambda_1^2\lambda_2^2}{s(1-c)}\right)\left(\Delta(\lambda_2)-\Delta(\lambda_1).\frac{\lambda_2^2(\lambda_1^2-2s)}{\lambda_1^2(\lambda_2^2-2s)}\right)\right\}=3 \tag{22}$$

$$a_2=\frac{1}{2}c_2'\left(1-\frac{\lambda_1^2\lambda_2^2}{s(1-c)}\right)\left(\Delta(\lambda_2)-\Delta(\lambda_1).\frac{\lambda_2^2(\lambda_1^2-2s)}{\lambda_1^2(\lambda_2^2-2s)}\right) \tag{23}$$

$$b_2'=-c_2'\frac{\lambda_2^2(\lambda_1^2-2s)}{\lambda_1^2(\lambda_2^2-2s)} \tag{24}$$

$$b_1=a_2\frac{\lambda_1^2\lambda_2^2}{2s(1-c)} \tag{25}$$

$$a_1=2+b_1 \tag{26}$$

4. Pressure Distribution

Solving equation (2), internal and external pressures are given by

$$P_i=\frac{2b_2r\cos\theta}{Da(1-c)}=2b_1r\cos\theta \tag{27}$$

$$P_e=\frac{2b_1\cos\theta}{r^2} \tag{28}$$

5. Drag on the Sphere

Drag on the sphere due to external flow is given by

$$\text{Drag } D_e = \pi U_\infty \int \varpi^3 \frac{\partial}{\partial n}\left(\frac{\mu E^2 \psi_e}{\varpi^2}\right) ds \quad \varpi = r\sin\theta,\ ds = rd\theta \text{ and } n = r \text{ for sphere} \quad (29)$$

The drag on the sphere due to the external flow reduces to

$$D_e = \frac{32\pi}{3} b_1 \mu a U_\infty \quad (30)$$

Hence non-dimensional drag=$D=D_e/6\pi\mu a U_\infty = 16b_1/9$

6. Results and Discussion

In Figure 2, the variation of stream function ψ for different values of couple stress parameters is presented. As s is increasing, the values of ψ are decreasing. In the limiting case as $s \to \infty$ we get a viscous fluid case. Hence we can conclude that the stream function for viscous fluids will have values less than the values of micropolar fluid.

In Figure 3, the variation of stream function ψ for different values of θ is presented. As θ values increase, the values of ψ also increase. From the figure, we observe that at the pole ($\theta = \pi/2$) the stream function is maximum. On the axis $\theta=0$ or π the stream function value is zero, i.e., on the equator, there will be a stagnation point.

In Figure 4, the variation of stream function ψ for different values of cross viscosity parameter c is presented. As c values are increasing, the values of ψ are also increasing. We observe from Figure 4 that as c tends to zero, the values of stream function decrease. As c tends to zero, we get viscous fluid. Hence we observe that (when the internal fluid is also viscous fluid) stream function for viscous fluid will have values less than stream function of micro-polar fluid.

In Figure 5, the variation of stream function ψ for different values of Da (Porosity parameter) is presented. As Da values are increasing, the values of ψ are decreasing.

In Figures 6, 7, and 8, contour graphs of Stream functions are shown for different values of s, Da, and c by keeping other parameters constant. In all the flows, we observe that the spherical region is created outside the porous sphere and fluid circulates at the top and bottom poles due to the external flow. The flow pattern is almost unchanged with the effects of s and Da as they increase in value. But as c increases, the outside formed sphere comes near the porous sphere and the flow internal to the porous sphere comes to zero, i.e., as c increases, the porous sphere will act as an impermeable sphere. This is a useful observation. As the micro-rotation increases, the internal flow decreases to flow inside the porous sphere.

From Figures 9 and 10, we observe that the variation of couple stress parameters on the drag is negligible. As c increases, drag numerically increases. As c increases, drag numerically decreases. As Da is increasing, we observe a drastic decrease in drag at small values of c. This may be because as the Darcy number increases, the porosity increases, and hindrance for the flow decreases and hence drag decreases. This is the usual observation in any flow past a porous medium. But here, the effect of micro-rotation parameter c is also present to decrease the drag.

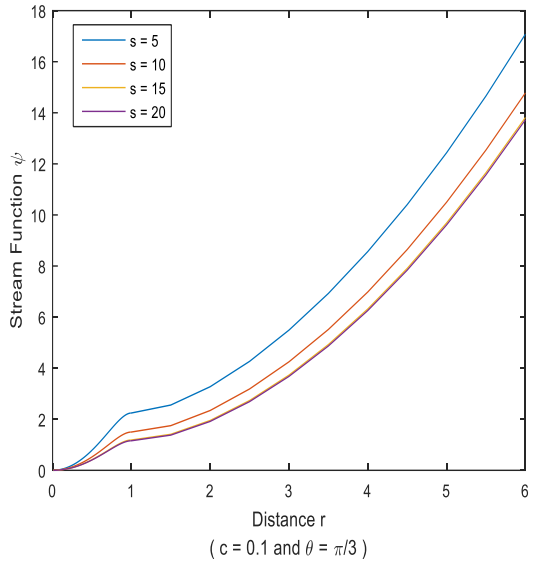


Figure 2. Distance r vs. Stream Function ψ .

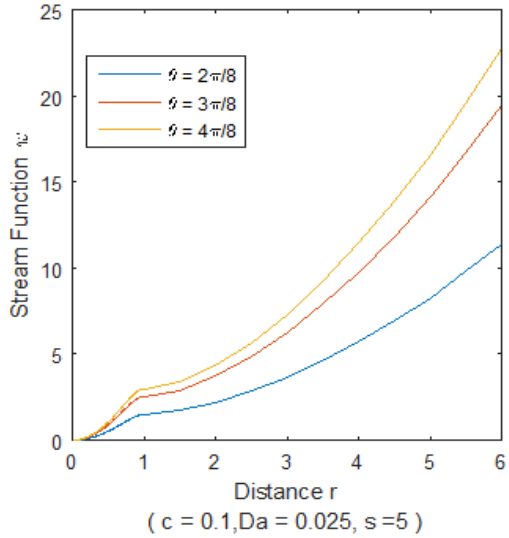


Figure 3. Distance r vs. Stream Function ψ .

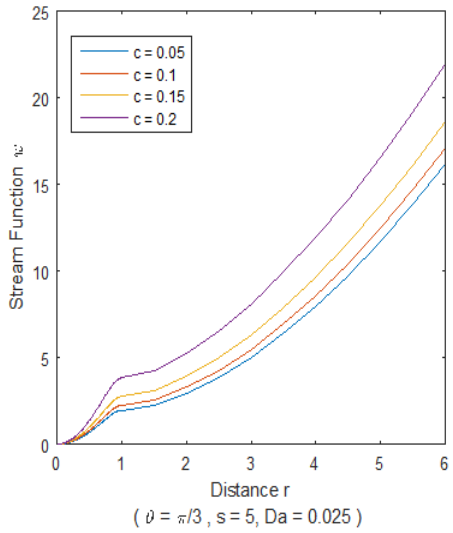


Figure 4. Distance r vs. Stream Function ψ .

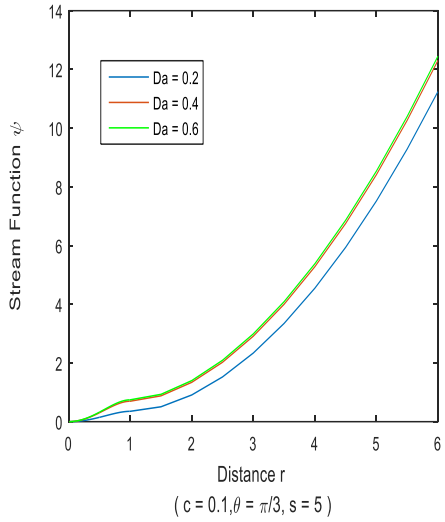
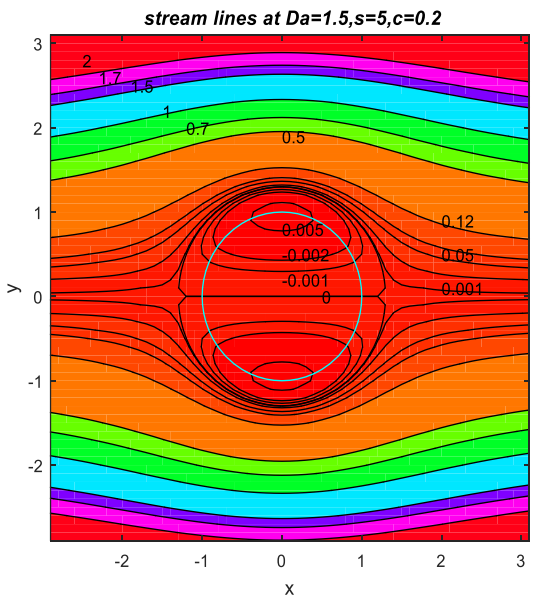
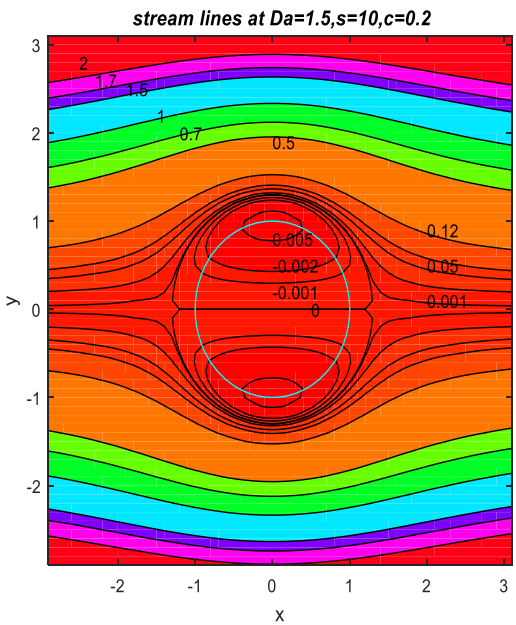


Figure 5. Distance r vs. Stream Function ψ .





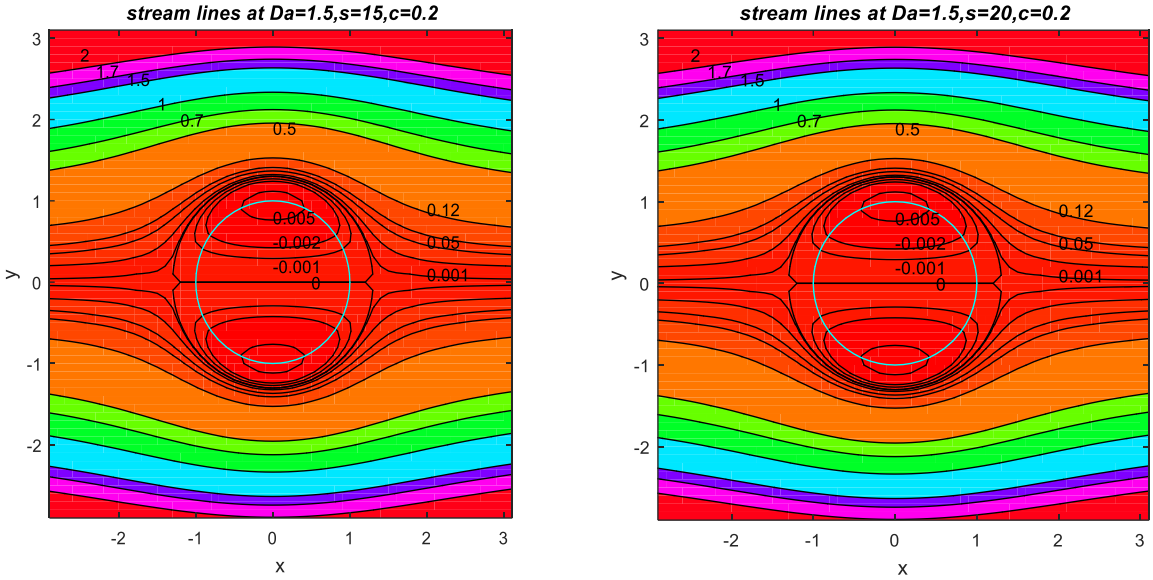


Figure 6. Contours of Stream function for Different values of s.

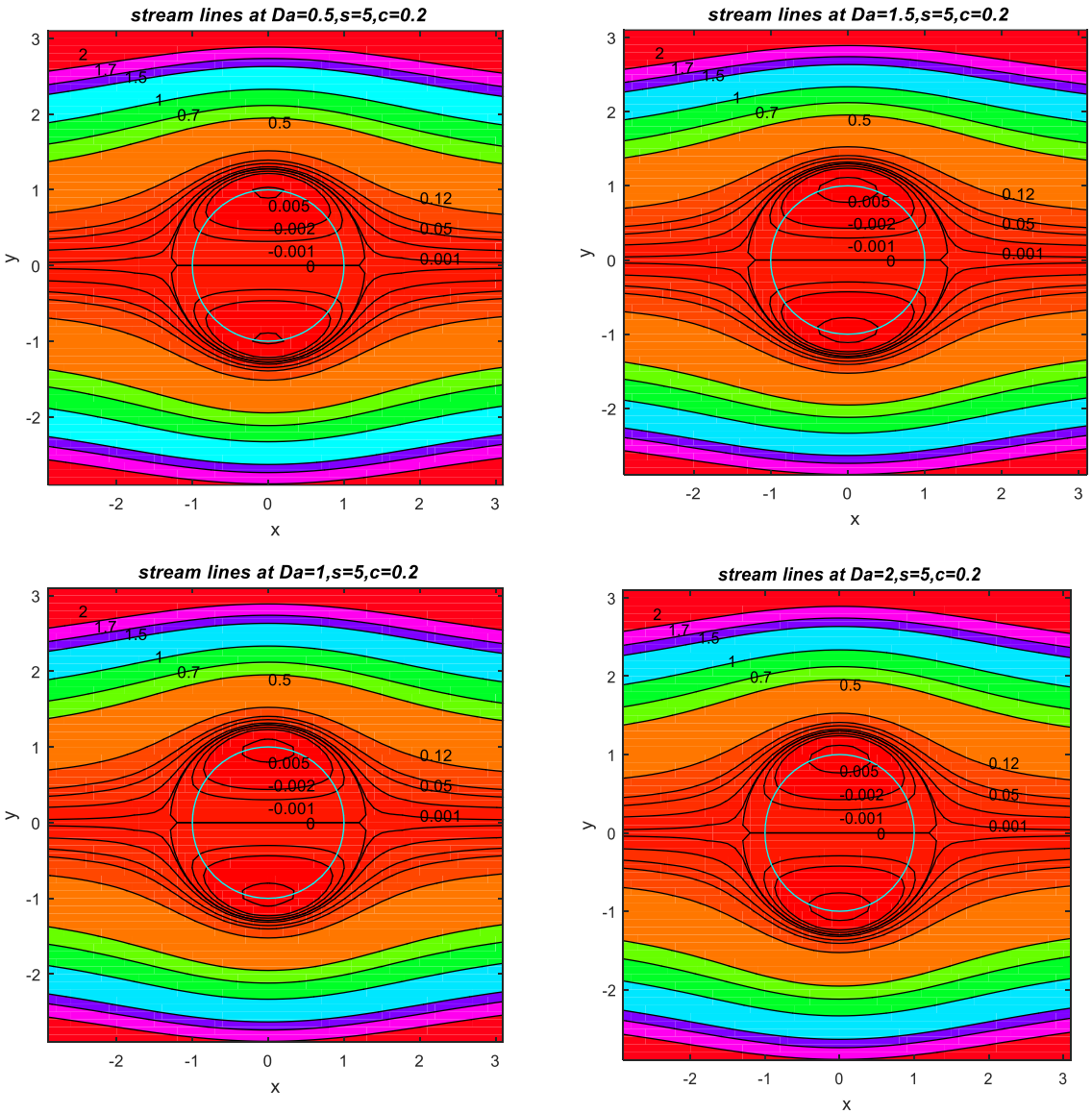


Figure 7. Contours of Stream function for Different values of Da.

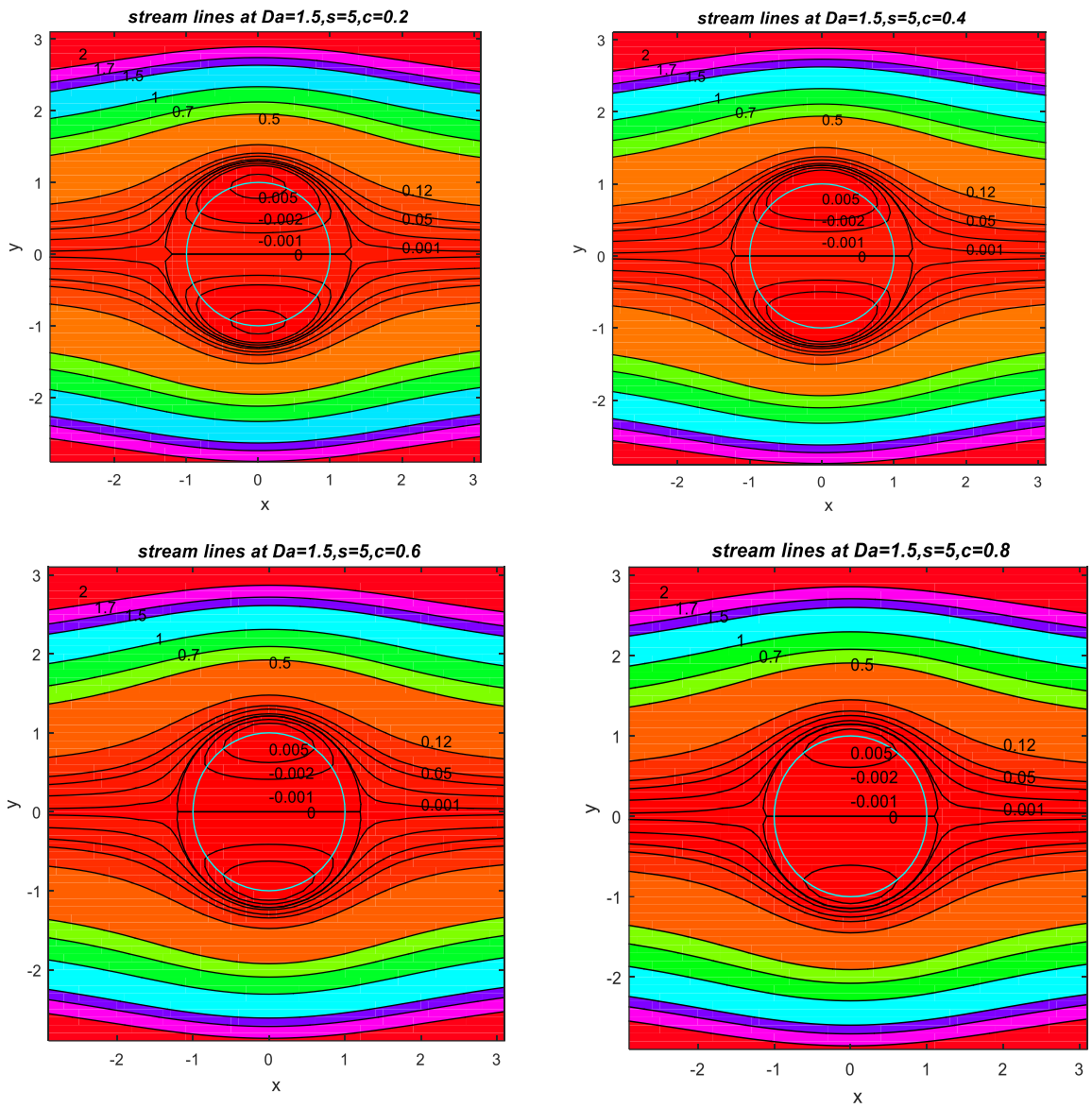


Figure 8. Contours of Stream function for Different values of c.

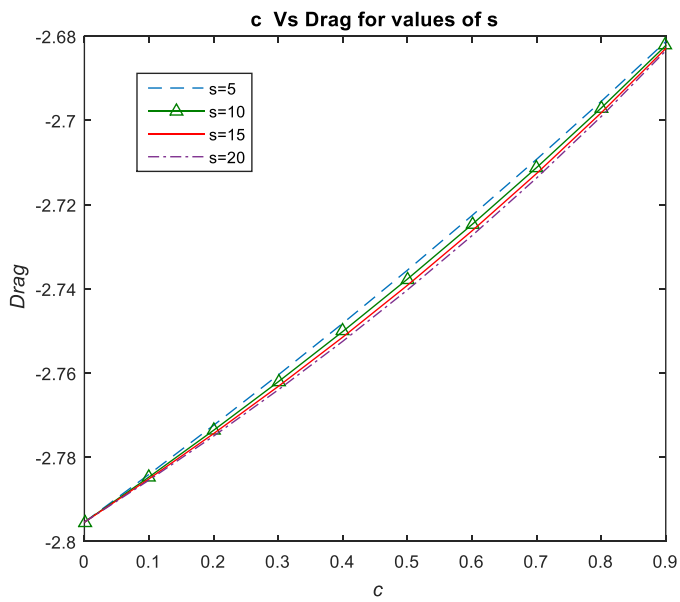


Figure 9. Cross viscosity vs. drag for various values of s.

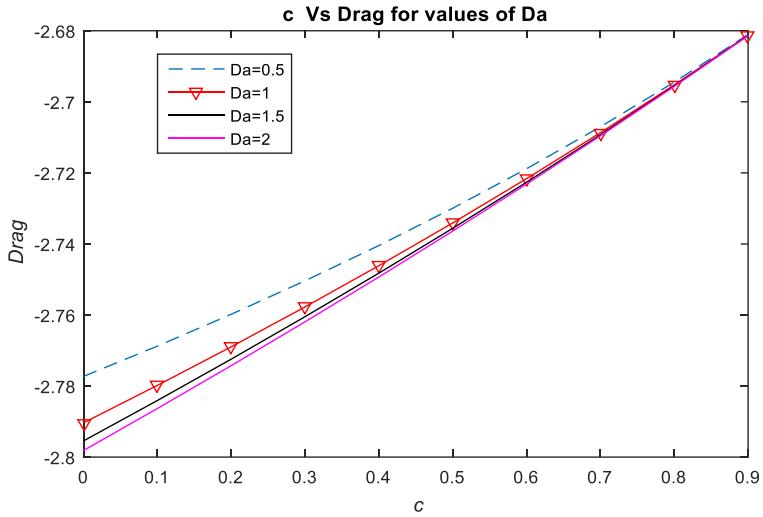


Figure 10. Cross viscosity vs. drag for various values of Da.

8. Conclusions

This research paper considers the uniform flow of a viscous fluid past a fixed porous sphere of radius *a*. The solution of the differential equation is calculated by the method of the superposition principle. The important observations of the above study are given below: As θ values increase, the values of the stream function ψ also increase; At the pole, the stream function is maximum; As cross viscosity parameter values increases, the value of stream function also increases; As Porosity parameter values increases, the value of stream function decreases; In the limiting case, as the cross-viscosity parameter increases, the porous sphere will be acting as an impermeable sphere.

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Conflicts of Interest

The authors claim that they are unaware of the competitive financial benefits or personal relationships that appear to affect the work reported in this paper.

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